úterý 20. října 2020 13:43

1) $M = \left\{ \frac{2n}{n^2+1} \mid n \in \mathbb{N} \right\}$

$\sup M = \max M = 1 \quad (n=1)$

$\inf M = 0 \quad \min M$ neexistuje

$2n \leq n^2+1 \iff 0 \leq n^2-2n+1 = (n-1)^2$

$0 < \frac{2n}{n^2+1} \leq 1$

$\exists n \in \mathbb{N}: \frac{2n}{n^2+1} < \varepsilon \;?$ $\quad \frac{1}{n} < \frac{2n}{n^2+1} < \frac{2}{n} < \frac{1}{\varepsilon}$ $\quad n > \frac{1}{\varepsilon}$

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a) $\sup A = 1 = \sup B \qquad \sup A \cup B \neq 0 \;?$

0,2b

$A = [0,1] = B \qquad \sup \{A \cup B\} = \max\{\sup A, \sup B\}$

b) $\sup A = 1 = \sup B \qquad \sup A \cap B \neq 1 \qquad A \cap B \neq \emptyset$

0,3b

$A = [0,1)$
$B = \{0,1\}$ $\Rightarrow A \cap B = \{0\} \quad \sup(A \cap B) = 0$

$\{\}$

$\forall \varepsilon > 0 \; \exists \delta > 0 \; \forall x \in \mathbb{R}: 0 < |x-a| < \delta \Rightarrow |f(x)-L| < \varepsilon$

$a=1$ $\quad \lim_{x \to a} f(x) = L$

$L=1$

$\lim x^2 = 1$

$|f(x)-L| = |x^2-1| = |x-1| \cdot |x+1| \leq 3|x-1|$

$\boxed{x \in [0,2]} \quad \boxed{|f(x)-L| \leq 3|x-1|}$

$\boxed{\delta < \varepsilon} \quad |x-1| < \delta \Rightarrow |x^2-1| < \varepsilon$

$\lim_{x\to 1} x^2 = 1$ | $\boxed{\delta \le \frac{\varepsilon}{3}}$ $|x-1|<\delta \Rightarrow |x^2-1|<\varepsilon$

$\boxed{\delta<1}$

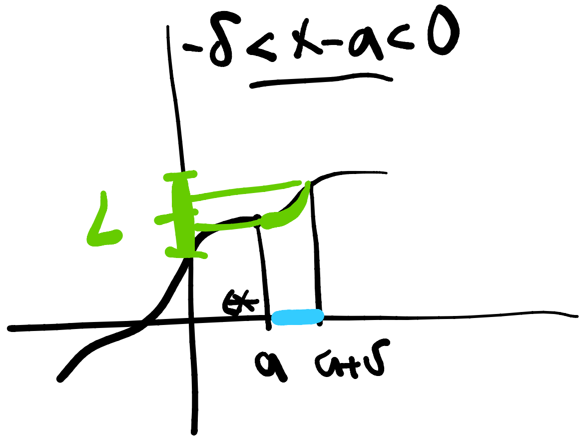
$\delta = \min\{\frac{\varepsilon}{3}, 1\} > 0$

$x\in[0,2] \Rightarrow |x+1| \le |2+1| = 3$

$\forall \varepsilon>0\ \exists \delta>0\ \forall x\in\mathbb{R}: 0< \underline{x-a}<\delta \Rightarrow |f(x)-L|<\varepsilon$

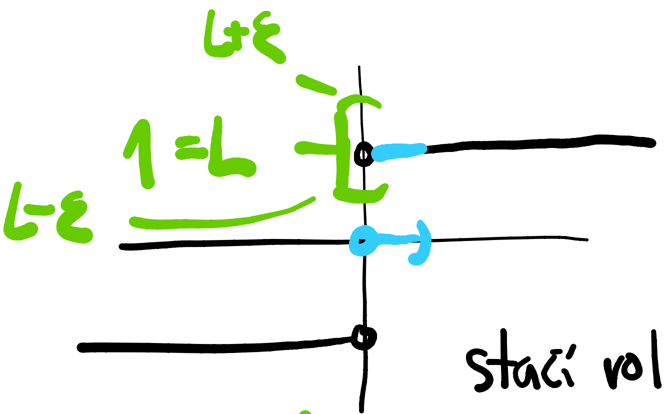
$-\delta < \underline{x-a}<0$

$\lim_{x\to a+} f(x) = L$



+ $\forall \varepsilon>0\ \exists \delta>0\ \forall x\in\mathbb{R}: 0< x-a<\delta \Rightarrow |f(x)-L|<\varepsilon$

− $\delta < x-a < 0$



$f(x) = \operatorname{sgn}(x) = \begin{cases}0 & x=0\\ -1 & x<0 \\ 1 & x>0\end{cases}$

$\lim_{x\to0^+} \operatorname{sgn} x = 1$

stačí volit $\underline{\delta = 301}$ (libovolné kladné)

$\forall \delta>0\ \forall x\in P^+(0,\delta): \operatorname{sgn}(x)=1$

$|1-1| = 0 < \varepsilon$ $\quad P^+(a,\delta) = (a,a+\delta)$

$\lim_{x\to0}\operatorname{sgn}(x)$ neexistuje $\quad \lim_{x\to0^-}\operatorname{sgn} x = -1$ $\quad P^-(a,\delta)=(a-\delta,a)$

$\left(\lim_{x\to a} f(x)=L\right) \iff \left[\left(\lim_{x\to a+} f(x)=L\right)\wedge\left(\lim_{x\to a-} f(x)=L\right)\right]$

$$\left(\lim_{x\to a} f(x)=L\right) \Leftrightarrow \left[\left(\lim_{x\to a+} f(x)=L\right) \wedge \left(\lim_{x\to a-} \dots\right)\right]$$

$$\lim_{x\to a} \frac{x^2-a^2}{x-a} = \lim_{x\to a} x+a = 2a$$

- $f=g$ na $P(a,\delta) \Rightarrow \left(\lim_{x\to a} f(x)=L\right) \Leftrightarrow \left(\lim_{x\to a} g(x)=L\right)$
- $\lim_{x\to a} f(x)+g(x) = A+B$ $\qquad A=\lim_{x\to a} f(x)$
- $\lim_{x\to a} f(x)\cdot g(x) = A\cdot B$ $\qquad B=\lim_{x\to a} g(x)$
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{A}{B}$ $\quad B\neq 0$
- f,g spojité v $\underline{a}$ $\Rightarrow$ $f+g,\ f\cdot g$ spojité v a

  $\frac{f}{g}$ spojitá v a, pokud $g(a)\neq 0$

$Ax+B$ spojitá $\rightarrow f(x)=x$ spojitá

$f(x)=C$ spojitá

$\rightarrow$ všechny polynomy jsou spojité

racionální funkce jsou spojité na D_f

$$\frac{3x\cdot x+x\cdot x\cdot x+3}{x\cdot x\cdot x-1} = \frac{3x^2+x^3+3}{x^3-1} \text{ spojitá mimo } 1$$

3a) $\lim \boxed{x^2-1} = \frac{0-1}{\dots} = 1$

3a) $$\lim_{x\to 0}\frac{\boxed{x^2-1}}{\boxed{2x^2-x-1}} = \frac{0-1}{2\cdot 0^2-0-1} = \underline{\underline{1}}$$

$0 \in D_f$

3b) $$\lim_{x\to 1}\frac{x^2-1}{2x^2-x-1} = \lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(2x+1)} = \lim_{x\to 1}\frac{x+1}{2x+1} = \underline{\underline{\frac{2}{3}}}$$

1 je kořen

$$\boxed{2x^2}-x-1 = \underbrace{2x^2-2x}_{\boxed{(x-1)\cdot 2x}}\ \underbrace{+2x-x-1}_{+(x-1)\cdot 1} = (x-1)\cdot(2x+1)$$

(3) - (8)

$\sqrt{x}$ je spojitá na $(0,\infty)$

$\sqrt{x^2+1}$ spojitá na $\mathbb{R}$

$a^2-b^2=(a-b)(a+b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a-b=\dfrac{a^2-b^2}{a+b}\cdot$

$\lim_{x\to a}$

$1 \quad - \quad 1 = 0$

(9) $$\lim_{x\to 0}\frac{\sqrt{1-2x-x^2}-(1-x)}{x} = \frac{(1-2x-x^2)-(1-x)^2}{x\left(\sqrt{1-2x-x^2}+(1-x)\right)}$$

$a=\sqrt{1-2x-x^2}$

$b=(1-x)$

$a-b\cdot$

$$=\lim_{x\to 0}\frac{1-2x-x^2-x^2+2x-1}{x\left(\sqrt{1-2x-x^2}+(1-x)\right)}$$

$$=\lim_{x\to 0}\frac{-2x^2}{x\left(\sqrt{1-2x-x^2}+(1-x)\right)}$$

$$=\lim \frac{-2x}{\quad} = 0$$

$$= \lim_{x\to 0} \frac{-2x}{\sqrt{1-2x-x^2} + (1-x)} = 0 =$$

(9)-(13)

úpravy $\Rightarrow$ rovnost na p. okolí $\rightarrow$ spojitost

- vytýkání kor. činitelů
- $a^n - b^n$

$\sqrt[3]{x^2 - x}$ spojitá na $\mathbb{R}$.