$$(1+\sin^2 x)^{|\sin x|} \to e^{|\sin x| \cdot \log(1+\sin^2 x)}$$

$$t=\log x \qquad dt=\frac{1}{x}dx \qquad \frac{A}{t}+\frac{B}{t+2}+\frac{C}{t-2}$$

Nevlastní limity

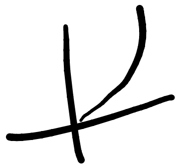
- $$\lim_{x\to+\infty} \frac{x^\alpha}{a^x} = \begin{cases} 0 & a>1 \\ +\infty & 0<a<1 \end{cases} \qquad \frac{x^2}{3^x} \to 0$$

$$\frac{x^\alpha}{\left(\frac{1}{3}\right)^x} = \frac{3^x}{\left(\frac{1}{x}\right)^\alpha}$$

$\alpha=1$

$$\lim_{x\to+\infty} \frac{\log x}{x^\alpha} = 0 \qquad \alpha>0$$

$$\lim_{x\to+\infty} x^\alpha = \begin{cases} +\infty & \alpha>0 \\ 1 & \alpha=0 \\ 0 & \alpha<0 \end{cases} \qquad \frac{2^x}{x^{50}} \to +\infty$$

x^2 

$$\lim_{x\to+\infty} \log x = +\infty \qquad x^{-2}=\frac{1}{x^2}$$

$$\lim_{x\to+\infty} \frac{\log x}{x^\alpha} = \lim_{x\to+\infty} \frac{\frac{1}{x}}{\alpha x^{\alpha-1}} = \lim_{x\to+\infty} \frac{1}{\alpha x^\alpha} = 0$$

$$\lim_{x\to+\infty} \frac{x^\alpha}{a^x} = 0 \qquad a>1$$

$$\lim_{x\to+\infty} \frac{\log x}{x^\alpha} = 0 \qquad \alpha>0$$

$$\lim \frac{3x^2+5x-2}{\ldots} = \lim \frac{\underset{\to 0}{\frac{3x^2}{3^x}} + \underset{\to 0}{\frac{5x}{3^x}} - \underset{\to 0}{\frac{2}{3^x}}}{\ldots}$$

$$\lim_{x\to+\infty} \frac{3x^4+3x-2}{\log x + 3^x + x^{15}} = \lim_{x\to+\infty} \frac{\frac{\dots}{3^x}+\frac{\dots}{3^x}-\frac{\dots}{3^x}}{\underbrace{\frac{\log x}{3^x}}_{\to 0} + \underbrace{1}_{\to 1} + \underbrace{\frac{x^{15}}{3^x}}_{\to 0}}$$

$$= 0$$

$$\lim_{x\to-\infty} \frac{x + x^2 - 4}{2x + \sqrt{x^4+x}} = \lim_{x\to+\infty} \frac{\underbrace{\frac{x}{x^2}}_{\to 0} + \underbrace{\frac{x^2}{x^2}}_{=1} - \underbrace{\frac{4}{x^2}}_{\to 0}}{\underbrace{\frac{2x}{x^2}}_{\to 0} + \sqrt{\frac{x^4+x}{x^4}}}$$

$$\sqrt{\frac{x^4+x}{x^4}} \to \sqrt{1+\underbrace{\frac{1}{x^3}}_{\to 0}}$$

$$x^2 = \sqrt{x^4}$$

$$= \frac{0+1+0}{0+\underbrace{\sqrt{1}}_{=1}} = 1$$

$$\frac{a^2-b^2}{a+b} = a-b$$

$$\lim_{x\to+\infty} \underbrace{\sqrt{1+x^2}}_{\substack{a\\ \to +\infty}} \ \overset{+}{-} \ \underbrace{x}_{\substack{b\\ \to \pm(+\infty)}} = \lim_{x\to+\infty} \frac{1+x^2-x^2}{\sqrt{1+x^2}+x}$$

$$= \lim_{x\to+\infty} \frac{1}{\sqrt{1+x^2}+x} = 0$$

$$\lim_{x\to+\infty} \sqrt{x+1}\left(\sqrt{1+x^2}-x\right) = \lim_{x\to+\infty} \frac{\sqrt{x+1}}{\sqrt{1+x^2}+x} = 0$$

$x \to$

$$\lim_{x\to+\infty} \frac{x}{\sqrt{1+x^2}+x} =$$

$$\frac{a^2-b^2}{a+b} = a-b$$

a b a^2 b^2

$x \to +\infty$

$$\sqrt{1+x^2} - x = \frac{1+x^2 - x^2}{\sqrt{1+x^2} + x}$$

a, b

$$\lim_{x\to+\infty} \frac{x}{\sqrt{1+x^2}+x} = \lim_{x\to+\infty} \frac{\frac{x}{x}}{\frac{\sqrt{1+x^2}}{\sqrt{x^2}} + \frac{x}{x}} = \lim_{x\to+\infty} \frac{1}{\sqrt{\frac{1}{x^2}+1}+1}$$

$\to 1$; 0, 1, 1

$$= \frac{1}{\sqrt{0+1}+1} = \frac{1}{2}$$

$$\lim_{x\to+\infty} \sqrt{1+x^2}$$

$f(x) = 1+x^2$ $\quad a=b=c=+\infty$

$g(x) = \sqrt{x}$

$f(x) \to +\infty = b$

$$\lim_{x\to+\infty} \sqrt{1+\frac{1}{x}}$$

$\frac{1}{x} \to 0$

---

l'Hospitalovo pravidlo

$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

$x=0$ $\quad 0 - 2(e^0-1) = 0$

$\frac{0}{0}$

$$\lim \frac{x(e^x+1) - 2(e^x-1)}{\ldots} =$$

$\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x)$

$\lim |g(x)| = +\infty$

$$\lim_{x\to 0} \frac{x(e^x+1)-2(e^x-1)}{x^3} =$$

$$\lim_{x\to a} |g(x)| = +\infty$$

$$\lim_{x\to 0} \frac{\overset{1+1}{e^x+1} + \overset{+0}{xe^x} - \overset{-2}{2e^x}}{3x^2} = \lim_{x\to 0} \frac{-e^x + e^x + xe^x}{6x}$$

$$\lim_{x\to+\infty} \frac{1}{x} = \lim_{x\to+\infty} \frac{0}{1} = 0$$

$$= \lim_{x\to 0} \frac{xe^x}{6x} = \frac{e^0}{6} = \underline{\underline{\frac{1}{6}}}$$

$$\lim_{x\to 0} \frac{1}{1+x} \cancel{=} \lim_{x\to 0} \frac{0}{0+1} = 0$$

Symboly $O, o, \sim$ $(\sim, \simeq)$

$f = o(g)$ $x\to a$ $\lim_{x\to a} \frac{f(x)}{g(x)} = 0$

$f = O(g)$ $x\to a$ $|f| \leq C\cdot|g|$ na $P(a,\Delta)$

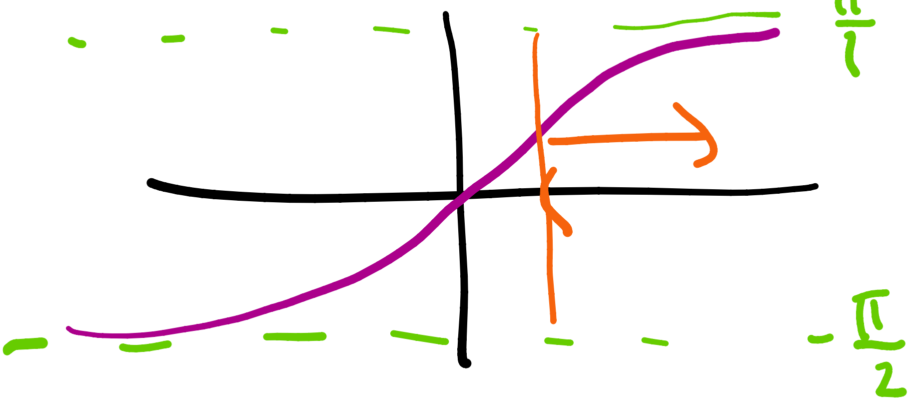
$f \sim g$ $x \to a$ $\underline{\lim_{x\to a} \frac{f(x)}{g(x)} = 1}$ $(\in \mathbb{R}\setminus\{0\})$

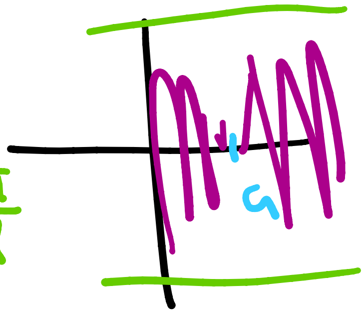
$\simeq$ $=1$

$f = O(1)$ $x\to a$ $|f| \leq C\cdot 1$ na $P(a,\Delta)$

$f = O(1) \quad x \to a$ $\qquad |f| \le C$

$\arctan x = O(1) \quad x \to +\infty$





- $f(x) = x(e^x+1) - 2(e^x-1)$

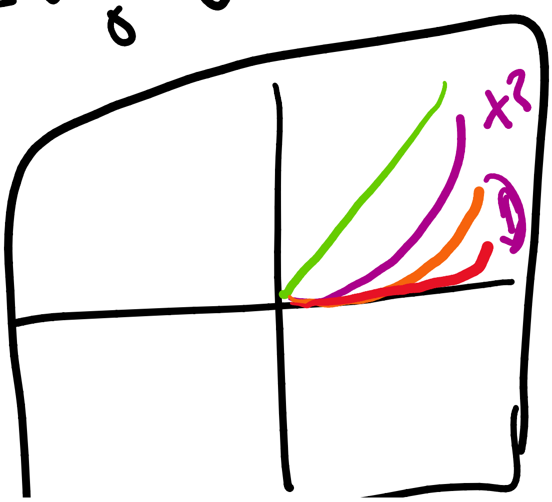
$g(x) = x^3$

$$f(0) = f'(0) = f''(0) = 0, \quad f'''(0) \neq 0$$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{1}{6} \qquad \lim_{x\to 0} \frac{f(x)}{\frac{1}{6}g(x)} = 1$$

$$f \sim \frac{1}{6} g \quad x \to 0$$

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0} \underbrace{x}_{\downarrow 0} \cdot \underbrace{\frac{f(x)}{x^3}}_{\downarrow \frac{1}{6}} = 0 \cdot \frac{1}{6} = 0$$



$$f(x) = o(x^2) \quad x \to 0$$

$x^? \quad x$ $\qquad a = ?$

$a = ?$

$$\frac{1+x}{1+x^4} \sim x^a \qquad x \to \infty$$

$1+x \sim x$, $\quad x^4 \sim 1+x^4$

$$\sim \frac{x}{x^4} = \frac{1}{x^3}$$

$$f_1 \sim g_1 \quad f_2 \sim g_2$$

$$\frac{f_1}{f_2} \sim \frac{g_1}{g_2} \qquad f_1 f_2 \sim g_1 g_2$$

$$\lim_{x\to\infty} \frac{\frac{1+x}{1+x^4}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{x^4+x}{1+x^4} = \lim_{x\to\infty} \frac{1+\overset{\to 0}{\frac{1}{x^3}}}{\underset{0\leftarrow}{\frac{1}{x^4}}+1} = \underline{1}$$

$$e^x - \cos x \sim x^a \qquad x \to 0$$

$$\lim_{x\to 0} \frac{e^x - \cos x}{x^a} = 1$$

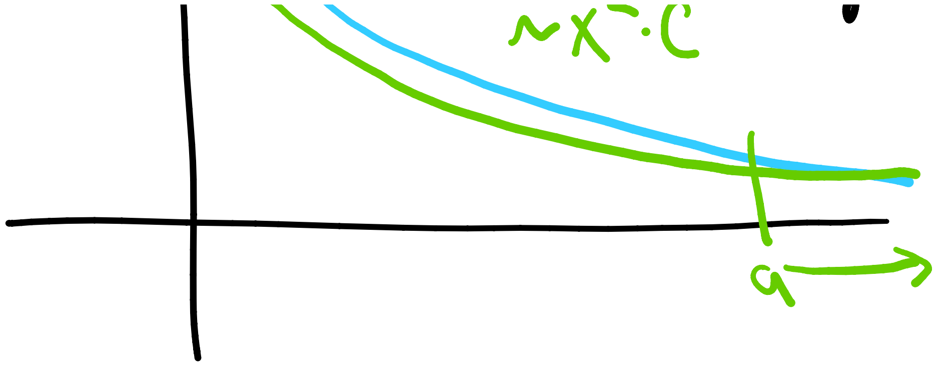
$a = 1$

$e^x - \cos x \to 0$

$e^x + \sin x \to 1 \neq 0$

$$\lim_{x\to 0} \frac{e^x - \cos x}{x} = 1 \qquad \frac{\overset{\to 1}{e^x - 1}}{x} + \frac{\overset{\to 0}{1-\cos x}}{x}$$

$e^x - \cos x - x \sim x^2 \cdot C \qquad f \sim g \cdot C$


a