Cvičení 17.3.

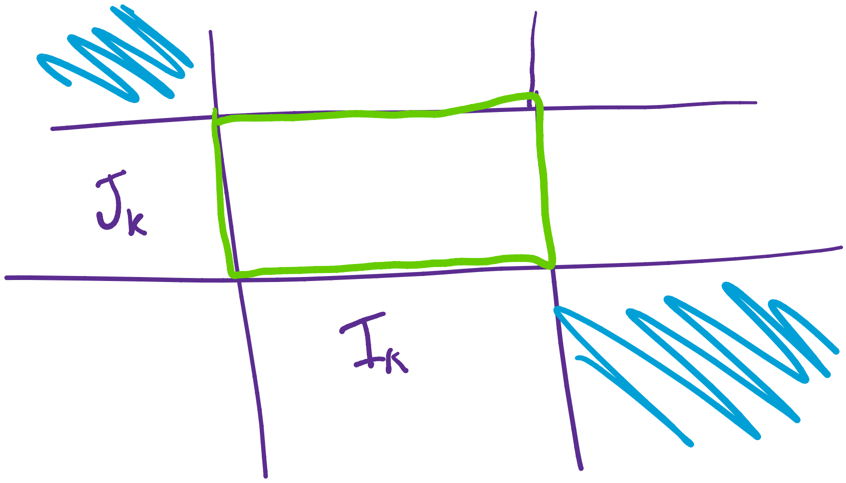
středa 17. března 2021 12:18

- $y' = f(x)\,g(y)$

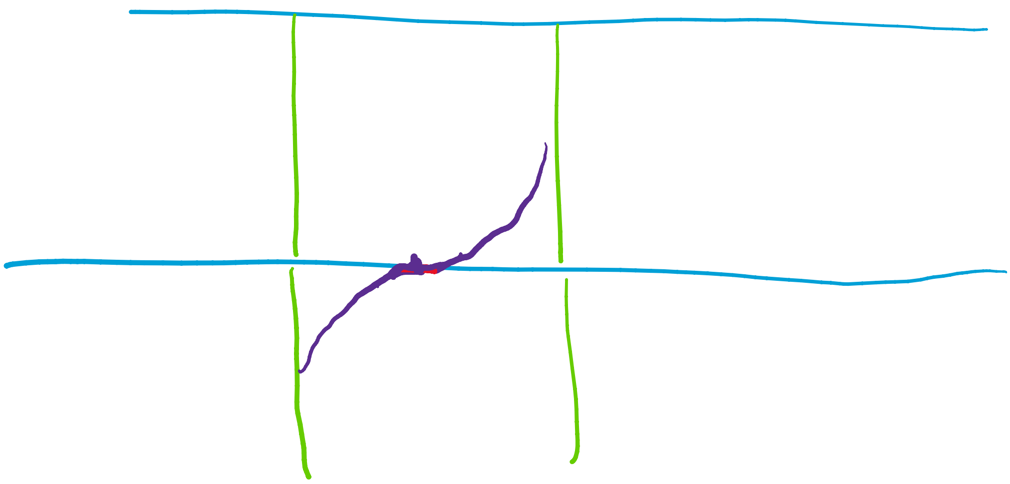
(1) definiční obor f $\boxed{I_k}$

(2) def. obor g, nulové body

(3) $g(c) = 0$ $\quad y = c \quad y' = 0$



$y = c$



$f(x) \cdot g(y)$ spojitá v (x_0, y_0)

$y' = 1 - y = f(x) \cdot g(y) \quad f(x) = 1 \quad g(y) = 1 - y$

(1) $D_f = \mathbb{R} \qquad I = \mathbb{R}$

(1) $D_f = \mathbb{R}$ $\quad I_1 = \mathbb{R}$

(2) $D_g = \mathbb{R}$, $g(y) = 0 \iff y = 1$ $\quad J_1 = (-\infty, 1)$, $J_2 = (1, \infty)$

(3) stacionární řešení $y = 1$

(4) $f(x) = 1 \to F(x) = x$ $\quad x \in I_1$

$g(y) = 1 - y$ $\quad \int \frac{1}{1-y}\,dy \overset{c}{=} -\log|1-y|$

$G(y) = -\log(1-y)$ $\quad y \in J_1 = (-\infty, 1)$

$G(y) = -\log(y-1)$ $\quad y \in J_2 = (1, \infty)$

(5) $\{\underset{x \in I_k}{x} : F(x) + C \in G(J_k)\} = \{x \in \mathbb{R} : F(x) + C \in \mathbb{R}\}$

$$\left.\begin{aligned} \lim_{y \to -\infty} G(y) &= \lim_{y \to -\infty} -\log(1-y) = -\infty \\ \lim_{y \to 1-} G(y) &= \lim_{y \to 1-} -\log(1-y) = +\infty \end{aligned}\right\} G(J_1) = \mathbb{R}$$

$$y_1(x) = G^{-1}(F(x) + C) = 1 - e^{-(x+C)}$$

$t = -\log(1-y)$ $\quad e^{-t} = 1 - y \to y = 1 - e^{-t}$

$$\lim_{y \to 1+} G(y) = \lim_{y \to 1+} -\log(y-1) = +\infty$$

$$\lim_{y \to +\infty} G(y) = \lim -\log(y-1) = -\infty$$

$$\lim_{y\to+\infty} G(y) = \lim_{y\to+\infty} -\log(y-1) = -\infty$$

$G(J_2) = \mathbb{R}$ $\qquad F(x)+C \in G(J_2) \qquad x\in\mathbb{R}$

$$y_2(x) = G^{-1}(F(x)+C) = 1+e^{-(x+c)}$$

$t = -\log(y-1) \qquad e^{-t} = y-1$

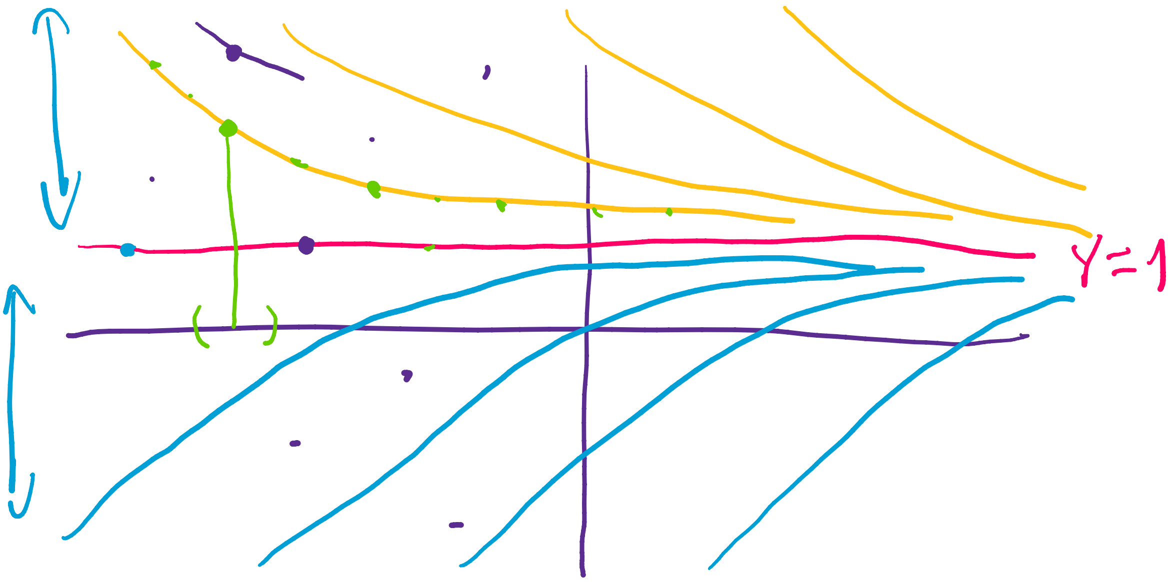
$G^{-1}(t) = 1+e^{-t}$

$y_0(x) = 1, \qquad x\in\mathbb{R} \quad \to \quad 1+0e^{-x} \qquad e^{-c}=D>0$

$y_1(x) = 1-e^{-(x+c)}, \quad x\in\mathbb{R} \quad \to \quad 1-De^{-x}$

$y_2(x) = 1+e^{-(x+c)}, \quad x\in\mathbb{R} \quad \to \quad 1+De^{-x}$

$\tilde{y}(x) = 1+De^{-x} \qquad x\in\mathbb{R}$



$y' = y^2-4$

$y' = 2x^2y^2 \qquad f(x) = 2x^2 \qquad g(y) = y^2$

(1) $I_1 = \mathbb{R}$

(2) $J_1 = (-\infty, 0), \quad J_2 = (0, \infty)$

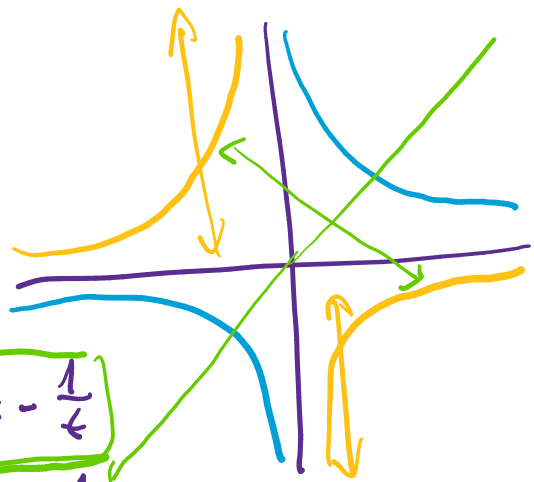
• (3) $y_0(x) = 0$

(4) $f(x) = 2x^2 \qquad \underline{F(x) = \frac{2}{3}x^3} \qquad x \in \mathbb{R} = I_1$

$g(y) = y^2 \qquad \int \frac{1}{y^2} \overset{c}{=} -\frac{1}{y}$

$G(y) = -\frac{1}{y} \qquad y \in (-\infty, 0)$

$G(y) = -\frac{1}{y} \qquad y \in (0, \infty)$



(5) $\underline{\{x : F(x) + C \in G(J)\}}$

$G(J_1) = (0, \infty) \rightarrow \boxed{G^{-1}(t) = -\frac{1}{t}}$

$G(J_2) = (-\infty, 0) \rightarrow \boxed{G^{-1}(t) = -\frac{1}{t}}$

$J_1 \rightarrow \quad F(x) + C \in (0, \infty) \iff \frac{2}{3}x^3 + C \in (0, \infty)$

$\iff \frac{2}{3}x^3 \in (-C, \infty) \iff x^3 \in (-\frac{3}{2}C, \infty)$

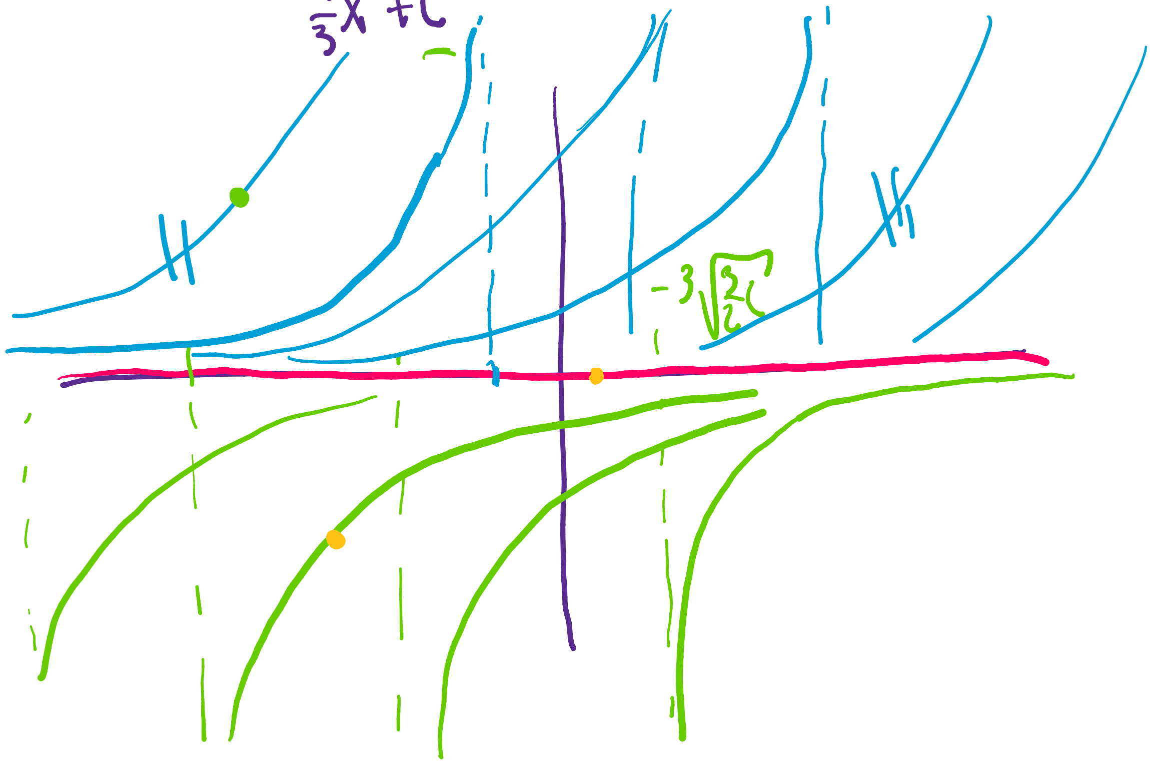
$\iff x \in \left(-\sqrt[3]{\frac{3}{2}C}, \infty\right)$

$\iff x \in \left(-\sqrt[3]{\frac{3}{2}C}, \infty\right)$

$J_2 \to \quad \frac{2}{3}x^3 + C \in (-\infty, 0) \iff \frac{2}{3}x^3 \in (-\infty, -C)$

$\iff x^3 \in \left(-\infty, -\frac{3}{2}C\right) \iff x \in \left(-\infty, -\sqrt[3]{\frac{3}{2}C}\right)$

- $y_1(x) = -\dfrac{1}{\frac{2}{3}x^3 + C}, \quad x \in \left(-\sqrt[3]{\frac{3}{2}C}, \infty\right)$
- $y_2(x) = -\dfrac{1}{\frac{2}{3}x^3 + C}, \quad x \in \left(-\infty, -\sqrt[3]{\frac{3}{2}C}\right)$



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(4) $y' = y^{\frac{3}{3}}$ $\qquad f(x) = 1 \quad g(y) = y^{\frac{2}{3}}$ $\qquad$ Sg

(1) $I = \mathbb{R}$

(2) $J_1 = (-\infty, 0) \quad J_2 = (0, \infty)$

(3) $y_0 = 0$

(4) ...

(3) $y_0 = 0$

(4) $F(x) = x$

$$\int \frac{1}{y^{\frac{2}{3}}} = \int y^{-\frac{2}{3}} = 3y^{\frac{1}{3}} \rightarrow$$

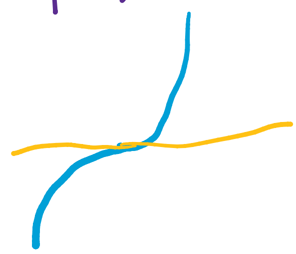
- $G(y) = 3y^{\frac{1}{3}} \qquad y \in (-\infty, 0)$
- $G(y) = 3y^{\frac{1}{3}} \qquad y \in (0, \infty)$

(5) $F(x) + C \in G(J)$ — $(-\infty, 0)$ / $(0, \infty)$

$$x + C \in (-\infty, 0) \iff x \in (-\infty, -C)$$

$$\in (0, \infty) \iff x \in (-C, \infty)$$

$$t = 3y^{\frac{1}{3}} \qquad y = \frac{1}{27} t^3$$

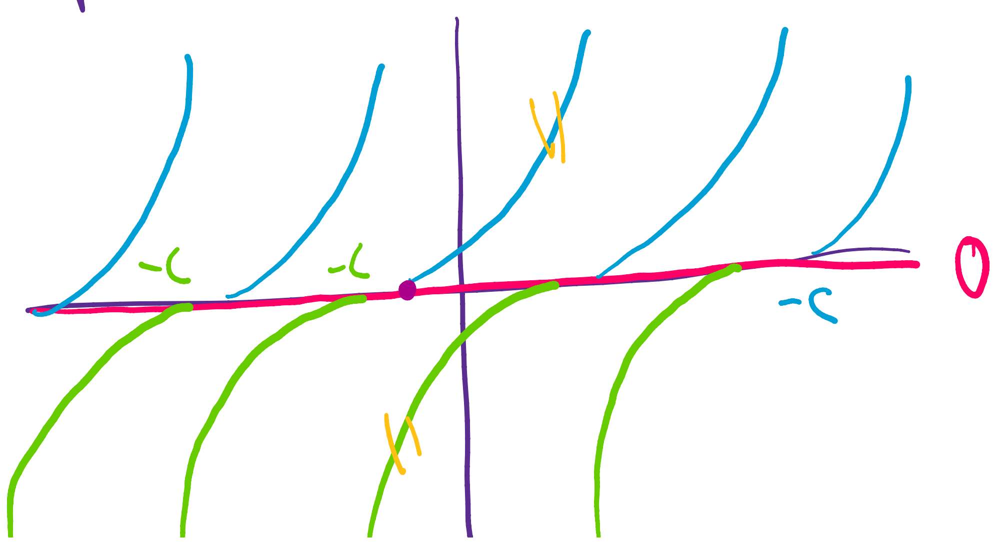


$$G^{-1}(F(x) + C) = \frac{1}{27}(x + c)^3 \searrow 0$$

$x \in (-\infty, -C)$

$x \in (-C, \infty)$

před lepením

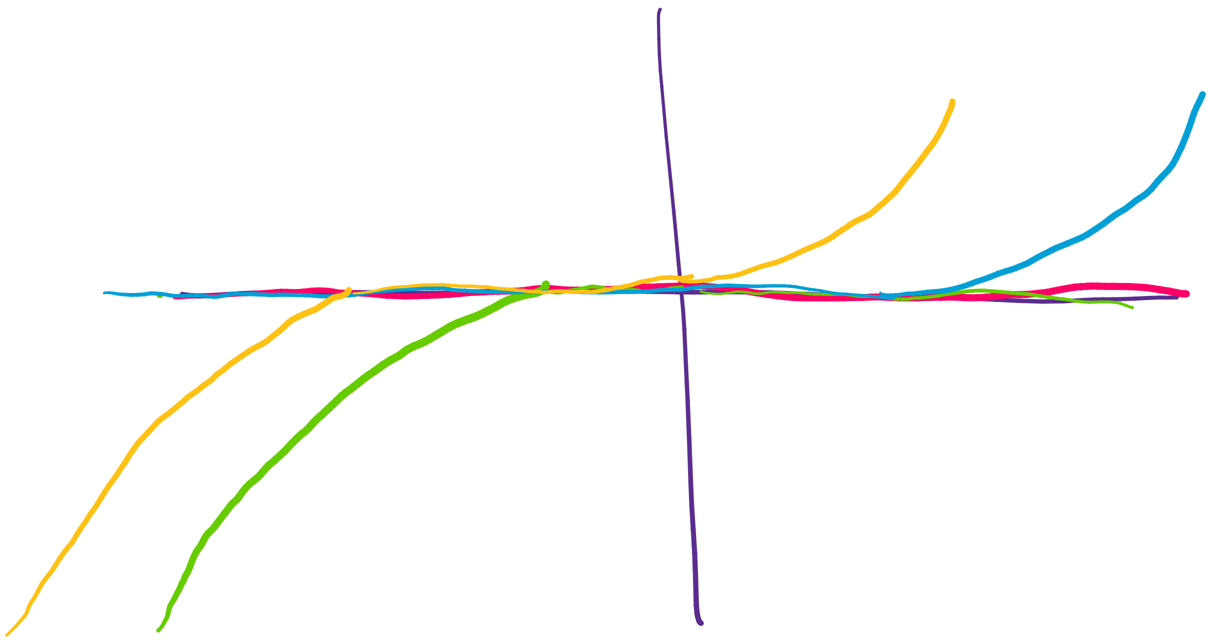


(6) • $y_0(x) = 0$

$$\tilde{y}_1(x) = \begin{cases} \frac{1}{27}(x+c)^3 & x \in (-\infty, -c) \\ 0 & x \in [-c, \infty) \end{cases}$$

$$\tilde{y}_2(x) = \begin{cases} \frac{1}{27}(x+c)^3 & x \in (-c, \infty) \\ 0 & x \in (-\infty, c] \end{cases}$$

• $$\tilde{y}_3(x) = \begin{cases} \frac{1}{27}(x+c)^3 & x \in (-\infty, -c) \\ 0 & x \in [-c, -D] \\ \frac{1}{27}(x+D)^3 & x \in (-D, \infty) \end{cases} \qquad -c < -D$$



(5) - (7) podobně

(8) $xy' + y = y^2$ $\qquad$ ~~$y' = f(x,y)$~~

$$\boxed{y' = \frac{y^2 - y}{x}} \qquad f(x) = \frac{1}{x} \qquad g(y) = y^2 - y$$

(1) $I_1 = (-\infty, 0)$ $\quad I_2 = (0, \infty)$

(2) $J_1 = (-\infty, 0]$, $\quad J_2 = (0,1)$, $\quad J_3 = (1, \infty)$

(3) $y_0(x) = 0 \quad y_0(x) = 1 \qquad x \in (-\infty, 0), (0, \infty)$

(4) $f(x) = \frac{1}{x}$

$$\boxed{\begin{array}{ll} F(x) = \log(-x) & x \in (-\infty, 0) \\ F(x) = \log(x) & x \in (0, \infty) \end{array}}$$

$g(y) = y^2 - y$

$$\int \frac{1}{y^2 - y} = \int \frac{-1}{y} + \frac{1}{y-1}\, dy$$

$$\stackrel{c}{=} -\log|y| + \log|y-1|$$

$$= \log\left|\frac{y-1}{y}\right|$$

$$\boxed{\begin{array}{ll} G(y) = \log \frac{y-1}{y} & y \in J_1 \\ G(y) = \log \frac{1-y}{y} & x \in J_2 \\ G(y) = \log \frac{y-1}{y} & y \in J_3 \end{array}}$$

$= (-\infty, 0) \qquad 0, +\infty$

$\lim_{y \to 0^+} G(y) = +\infty$

$\lim_{y \to 1^-} G(y) = -\infty$

$(1, \infty) \qquad -\infty, 0$

(5) $I \quad J \quad \ldots \quad = G(J)$

(5) I_1, J_1 $\quad F(x)+C \in G(J_1)$ •

• $\log(-x)+C \in (0,\infty) \Leftrightarrow \log(-x) \in (-C,\infty)$

$\Leftrightarrow -x \in (e^{-c},\infty) = x \in (-\infty, -e^{c})$

• $G^{-1}(F(x)+C) = \dfrac{1}{1-e^{\log(-x)+c}} = \boxed{\dfrac{1}{1+e^{c}x}}$

$t = \log\left(\frac{y-1}{y}\right) \qquad e^{t} = \frac{y-1}{y}$

$ye^{t} = y-1 \qquad y(1-e^{t}) = 1$

$$\boxed{G^{-1}(t) = \frac{1}{1-e^{t}}}$$

$I_1, J_2 \qquad G(J_2) = \mathbb{R}$

$I_1 = (-\infty,0) \qquad J_2 = (0,1)$

$t = \log\frac{1-y}{y} \qquad e^{t} = \frac{1-y}{y} \qquad ye^{t} = 1-y$

$y(1+e^{t}) = 1 \qquad y = \frac{1}{1+e^{t}}$

$G^{-1}(F(x)+c) = \dfrac{1}{1+e^{\log(-x)+c}} = \boxed{\dfrac{1}{1-e^{c}x}}$

$$\boxed{y_3(x) = \frac{1}{1+e^{c}x} \qquad x \in (-\infty, -e^{c})}$$

$$y_3(x) = \frac{\quad}{1+e^{c}x} \qquad x \in (-\infty, -e^{c})$$

$$y_4(x) = \frac{1}{1-e^{c}x} \qquad x \in (-\infty, 0)$$

$$y_5(x) = \frac{1}{1+e^{c}x} \qquad x \in (-e^{c}, 0)$$

$$\log(-x) + C \in (-\infty, 0) \iff \log(-x) \in (-\infty, -C)$$

$$-x \in (0, e^{-c}) \iff x \in (-e^{-c}, 0)$$