Cvičení 15.12.

úterý 15. prosince 2020 13:41

(1) $f(x) = x^3 - 6x^2 + 9x - 4 \qquad x \in \mathbb{R} \qquad \lim_{x\to\pm\infty} f(x) = \pm\infty \qquad R_f = \mathbb{R}$

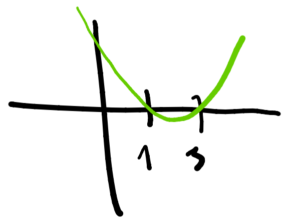
$f'(x) = 3x^2 - 12x + 9 = 3(1x^2 - 4x + 3) = 3(x-1)(x-3)$

$1\cdot 3 \quad (\pm 1, \pm 3)$

- $f'(x) = 0 \iff x = 1, 3$

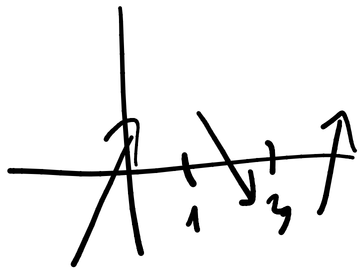
$f'(x) > 0 \qquad x \in (-\infty, 1) \cup (3, \infty)$

$f'(x) < 0 \qquad x \in (1, 3)$



f je rostoucí na $(-\infty, 1]$ a $[3, \infty)$

klesající na $[1, 3]$



1 je bodem lok. maxima

3 je bodem lok. minima

$f'' > 0$ na $U(a, \delta) \Rightarrow$ lok. min.

$f''(x) = 6x - 12 = 6(x-2)$

(4) $f(x) = \underline{x^2 - 4x + 6}$ // Polynom $\qquad x \in [-3, 10] = M$

globální ext.

globální ext.

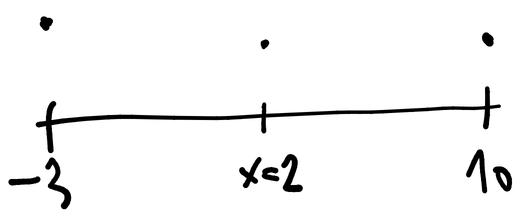
- f spojitá na $[-3,10] = M$ $\qquad f \in C([-3,10])$
- M je omezený uzavřený interval

$\Rightarrow$ f nabývá glob. minima i glob. maxima vzhledem k M.

x bodem glob. extrému $\rightarrow$ x bodem lok. extrému

$$f'(x) = 2x - 4 \quad \rightarrow \quad f'(x) = 0 \iff x = 2$$

$x^2 - 4x + 6$



$f(-3), f(2), f(10)$

$$f(-3) = 9 + 12 + 6 = 27$$

$$f(2) = 4 - 8 + 6 = 2 \quad \text{MIN}$$

$$f(10) = 100 - 40 + 6 = 66 \quad \text{MAX}$$

$$f(x) = \underline{e^{-x^2}} \qquad x \in \mathbb{R}$$

$$f'(x) = -2x\,e^{-x^2}$$

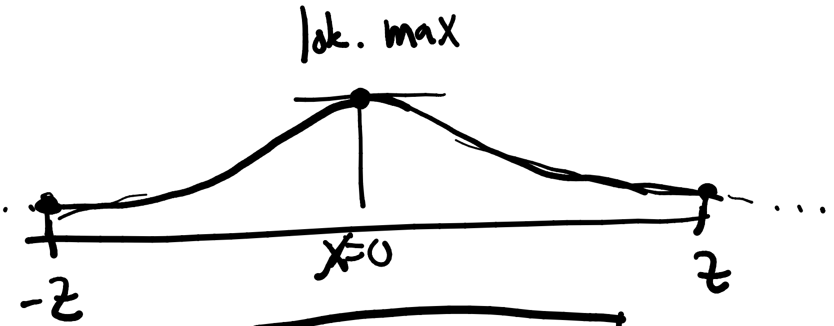
$f'(x) = 0 \iff x = 0$

$f'(x) < 0 \iff x > 0$

$f'(x) > 0 \iff x < 0$.

lok. max

$f'(x) > 0 \iff x < 0$



$f(0) = \underline{1}$

$$\boxed{\lim_{x \to \pm\infty} f(x) = 0}$$

f nabývá na $[-z, z]$ glob. extrémů

$\underline{f(x) \leq \frac{1}{z}}$ na $\underline{(-\infty, -z), (z, \infty)}$ $\quad f(x) > 0$

$R_f = (0, 1]$ $\quad \inf R_f = 0$

$\sup R_f = 1 = \max$

(1) $f(x) = \arcsin\left(\frac{2x}{x^2+1}\right)$

- $D_f = \mathbb{R} \rightarrow \frac{2x}{x^2+1} \rightarrow \mathbb{R}$

$\arcsin x \rightarrow [-1, 1]$

$-1 \leq \frac{2x}{x^2+1} \leq 1 \quad \Leftrightarrow -x^2 - 1 \leq 2x \leq x^2+1$

$-x^2-2x-1 \leq 0 \leq x^2-2x+1$

$-(x+1)^2 \qquad (x-1)^2$

$x=-1 \quad = \quad x=+1$

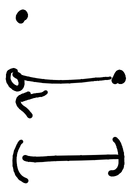
- f spojitá na D_f

- ~~sudá~~, lichá, ~~periodická~~

$f(-x) = -f(x)$

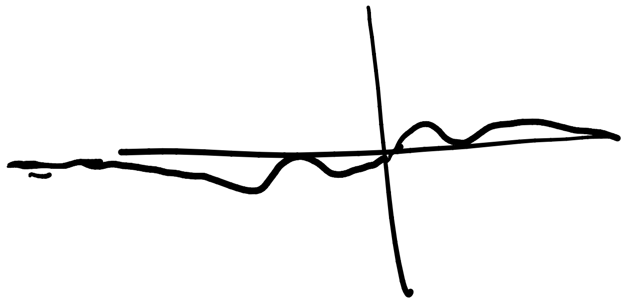
$$\arcsin\left(\frac{2(-x)}{(-x)^2+1}\right) = \arcsin\left(-\frac{2x}{x^2+1}\right) = -\arcsin\left(\frac{2x}{x^2+1}\right)$$

- limity v krajních bodech D_f



$$\lim_{x\to\pm\infty} \arcsin\left(\frac{2x}{1+x^2}\right) = 0$$

$\frac{2x}{1+x^2} \to 0$



$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

→ • $f'(x) = ?$

$$\left(\arcsin\left(\frac{2x}{1+x^2}\right)\right)' = \frac{2(1+x^2)-4x^2}{(1+x^2)^2}\cdot\frac{1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}}$$

$1+2x^2+x^4-4x^2=$

$$= \frac{2-2x^2}{(1+x^2)^2}\cdot\frac{1+x^2}{\sqrt{(1+x^2)^2-4x^2}} = 2\,\frac{1-x^2}{1+x^2}\,\frac{1}{\sqrt{1-2x^2+x^4}}$$

$(1-x^2)^2$

$\boxed{\frac{x}{|x|} = \operatorname{sgn} x}$

$= 2\cdot\frac{1-x^2}{\ldots}\cdot\frac{1}{\ldots} = 2\,\boxed{\operatorname{sgn}(1-x^2)}\,\frac{1}{\ldots}$

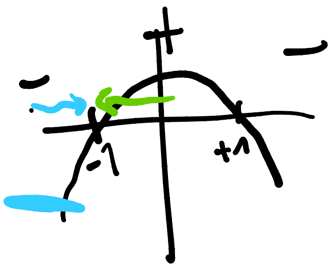
$$= 2\cdot \frac{1-x^2}{|1-x^2|}\cdot\frac{1}{1+x^2} = \underline{\underline{2\,\boxed{\operatorname{sgn}(1-x^2)}\,\frac{1}{1+x^2}}}$$

$(1-x^2)$

$\underline{\underline{x\in\mathbb{R}\setminus\{\pm1\}}}$

- limity f' v krajních bodech

f spojitá na $\mathbb{R}$



$$f'_-(-1)=\lim_{x\to-1-} f'(x) = -2\,\frac{1}{1+x^2} = -1$$

$$f'_+(-1)=\lim_{x\to-1+} f'(x)=2\,\frac{1}{1+x^2}=+1$$

$$\boxed{f'_\pm(1)=\mp1 \qquad f'_\pm(-1)=\pm1}$$

$$\lim_{x\to\pm\infty} f'(x)=0$$

- monotonie a extrémy

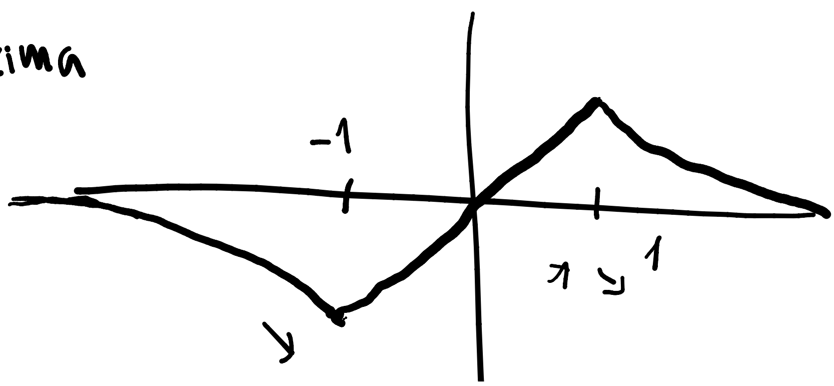
$$f'(x)=2\operatorname{sgn}(1-x^2)\cdot\underbrace{\frac{1}{1+x^2}}_{>0}$$

$[-1,1]$

	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
f'	−	+	−
f	↘	↗	↘

-1 bod lok. minima

1 bod lok. maxima



- f''

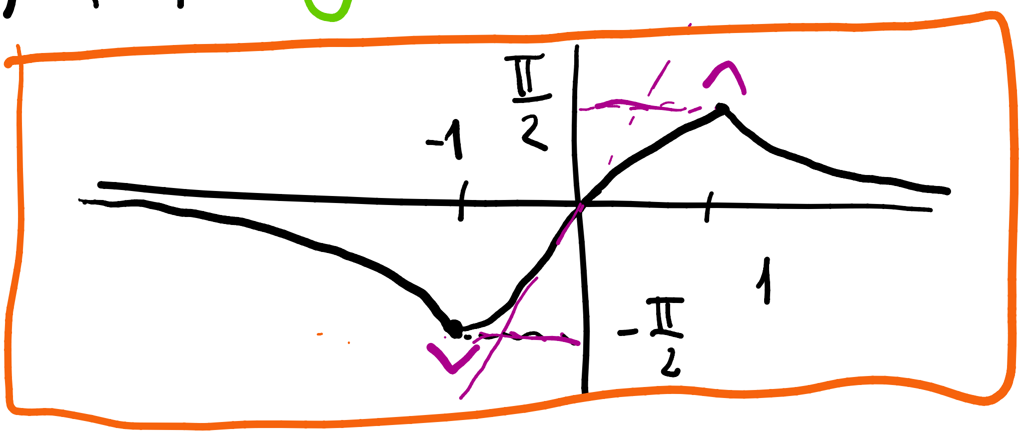
$$f'(x) = 2\,\mathrm{sgn}(1-x^2) \cdot \frac{1}{1+x^2}$$

$x=0$

$$f''(x) = 2\,\mathrm{sgn}(1-x^2) \cdot \frac{-2x}{(1+x^2)^2} \qquad x \in \mathbb{R} \setminus \{\pm 1\}$$

	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
$-x$	+	+	−	−
$\mathrm{sgn}(1-x^2)$	−	+	+	−
$f''(x)$	−	+	−	+
	∩	∪	∩	∪

0 inflexní bod



- asymptoty $y=0$ $\qquad f([-1,1]) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

- obor hodnot $\quad R_f = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R \quad = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R_{arcsin} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

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(2) $f(x) = (x-1)\, e^{\frac{x}{x+1}}$

- $D_f = \mathbb{R} \setminus \{-1\}$
- spojitá na D_f – skládám spojité funkce
- není sudá, lichá ani periodická
- limity v krajních bodech D_f $\pm\infty, -1$

$f(x) = \underset{-}{(x-1)}\, e^{\frac{x}{x+1}}$ $\to -1$, $\to +\infty$, 0^-

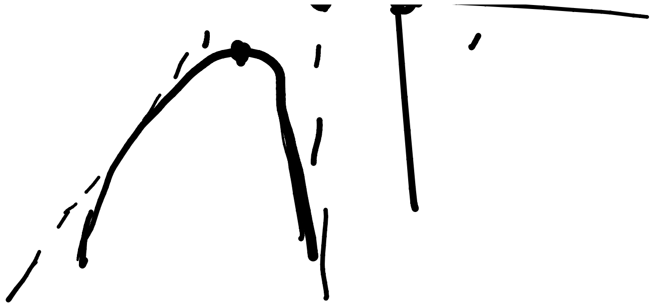
- $+\infty$
- $-\infty$
- $-1+$
- $-1-$

$\lim\limits_{x \to +\infty} f(x) = +\infty$

$\lim\limits_{x \to -\infty} f(x) = -\infty$

$\lim\limits_{x \to -1^+} f(x) = 0$

$\lim\limits_{x \to -1^-} f(x) = -\infty$



• $f'(x) = \dfrac{x(3+x)}{(1+x)^2}\, e^{\frac{x}{1+x}}$ $x \in D_f = \mathbb{R} \setminus \{-1\}$

$\lim\limits_{x\to-\infty} f'(x) = e = \lim\limits_{x\to+\infty} f'(x)$

$\lim\limits_{x\to-1^-} f'(x) = -\infty$ $\lim\limits_{x\to-1^+} f'(x) = 0$

• monotonie a extrémy

$x(3+x) = 0 \iff x = 0, -3$

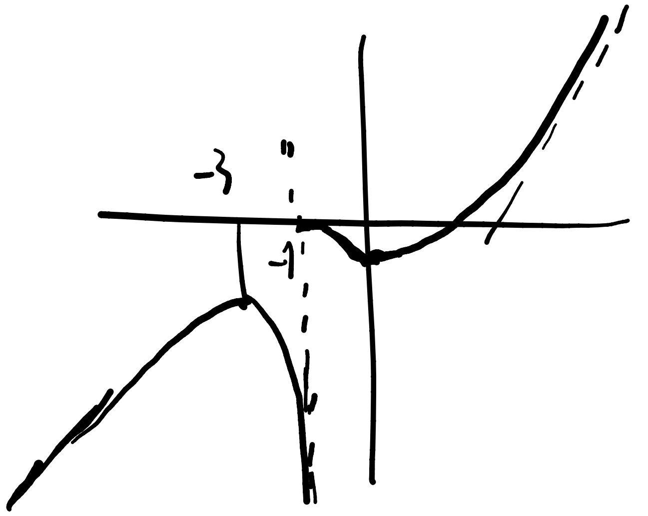
$-$ $+$

$+$ $+$

$(-\infty, -3)$	$(-3, -1)$	$(-1, 0)$	$(0, \infty)$
$+$	$-$	$-$	$+$
$\nearrow$	$\searrow$	$\searrow$	$\nearrow$

-3 bodem lok. maxima

0 bodem lok. minima



$$\bullet \; f''(x) = \frac{3+5x}{(1+x)^4} e^{\frac{x}{1+x}}$$

$(1+x)^4 > 0$, $e^{\frac{x}{1+x}} > 0$

$(-\infty, -1)$	$(-1, -\frac{3}{5})$	$(-\frac{3}{5}, \infty)$
$-$	$-$	$+$
$\frown$	$\frown$	$\smile$