úterý 13. října 2020 13:56

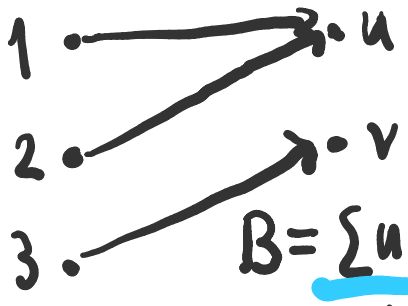
$B \subseteq Y$

$f: X \to Y$

$f(f^{-1}(B)) \overset{\supseteq}{\subseteq} B$

$x \in f(f^{-1}(B)) \Rightarrow x \in B$

$\left.\begin{array}{l} \exists \underbrace{y \in f^{-1}(B)}_{\Updownarrow \; f(y) \in B} : x = f(y) \end{array}\right\} \Rightarrow x = f(x) \in B$



$B = \{u, v\}$ $\quad f^{-1}(B) = \{1, 2\}$

$f(f^{-1}(B)) = f(\{1,2\}) = \{u\}$

$7 \mid 5^{2n+1} + 2^{2n+1}$ $\quad n \in \mathbb{N}$

1) $5^{2+1} + 2^{2+1} = 133$ ✓

2) $5^{2(n+1)+1} + 2^{2(n+1)+1} = \underset{21+4}{25} \cdot 5^{2n+1} + 4 \cdot 2^{2n+1}$

$= 4(\underbrace{5^{2n+1} + 2^{2n+1}}_{IP}) + \underset{\uparrow \; 3 \cdot 7 \cdot 5^{2n+1}}{21 \cdot 5^{2n+1}}$

$s = \sup M$ • $x \in M \Rightarrow x \le s$ $\qquad s \in M$

(y h.z. $M \Rightarrow y \ge s$)

• $\forall \varepsilon > 0\ \exists x \in M : x > s - \varepsilon$ $\qquad s > s - \varepsilon$

Archimedova vlastnost $\mathbb{R}$.

$$\forall x \in \mathbb{R}\ \exists n \in \mathbb{N} : n > x$$

1) $M = (0,1] = \{x : 0 < x \le 1\}$

$x \in M$ je maximum, pokud $\forall y \in M : x \ge y$

$x \in M$ je minimum, — || — : $x \le y$

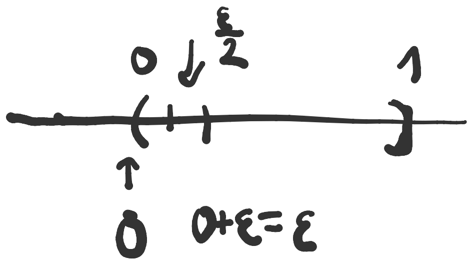
$1 \in M$, 1 je horní závora

$\Rightarrow 1 = \max M = \sup M$

0 je dolní závora

$\boxed{\varepsilon > 0}$ — $\boxed{\varepsilon < 1}$ $\rightarrow$

$\boxed{\varepsilon \ge 1}$ •



$x = \inf M$: • x je dolní závora

y $\qquad$ • $\forall \varepsilon > 0\ \exists y \in M : y < x + \varepsilon$ •

$0 < \boxed{\frac{\varepsilon}{2}} < 1$ $\qquad \frac{\varepsilon}{2} \in M$, $\frac{\varepsilon}{2} < \varepsilon$

$y = \boxed{\frac{1}{2}} \in M$ $\qquad y < \varepsilon$

$\rightarrow \quad y = \boxed{\frac{1}{2}} \in M \qquad y < \varepsilon$

0 je tedy infimum M.

$0 \notin M$ a tedy není minimem M

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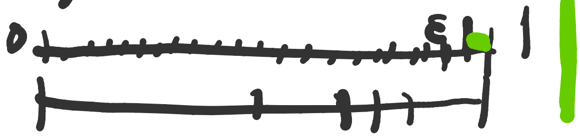
b) $M = \left\{ \frac{m}{m+n} : m, n \in \mathbb{N} \right\}$ $\qquad \mathbb{N} = 1, 2, \dots$

sup M, inf M ?

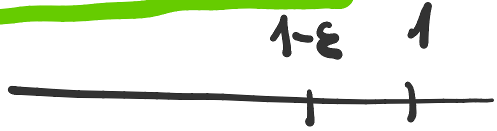


$0 < \frac{m}{m+n} < 1$ $\qquad n = 1 \rightarrow \frac{m}{m+1}$ $\qquad 1 - \frac{m}{m+1} \leq \frac{1}{m}$

$1 - \frac{3}{4} \leq \frac{1}{4}$

$m < m+n$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots$



$\frac{m}{m+1} > 1 - \varepsilon$

• $\forall \varepsilon > 0 \ \exists m \in \mathbb{N} : \boxed{\frac{m}{m+1} > 1 - \varepsilon}$ $\quad \checkmark \quad \forall x \in \mathbb{R} \ \exists n \in \mathbb{N} : n > x$

$$\frac{m}{m+1} > 1 - \varepsilon \iff m > (m+1)(1-\varepsilon)$$
$$\iff m > m(1-\varepsilon) + 1 - \varepsilon$$
$$\iff m\varepsilon > 1 - \varepsilon$$
$$\iff m > \frac{1-\varepsilon}{\varepsilon}$$

$1 \notin M$ tedy $1 = \sup M$, ale max M neexistuje.

neexistuje.

$M = \left\{ \frac{m}{m+n} : m,n \in \mathbb{N} \right\}$

$0 < \frac{m}{m+n}$

- $\forall \varepsilon > 0 \ \exists m,n \in \mathbb{N} : \frac{m}{m+n} < \varepsilon \quad (= 0 + \varepsilon)$

$m = 1 \to \frac{1}{1+n}$ $\qquad \exists n \in \mathbb{N} : n > \frac{1}{\varepsilon}$

$n > \frac{1}{\varepsilon} \ \boxed{\Longleftrightarrow} \ \left[\varepsilon > \frac{1}{n} > \frac{1}{n+1} \right] \qquad \varepsilon > \frac{1}{n+1}$

$0 = \inf M$, ale $\min M$ neexistuje

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c) $M = \{ n^2 - m^2 : n,m \in \mathbb{N} \}$

M není shora/zdola omezená ?

pokud s je horní závora,

potom $\quad n^2 - m^2 \leq s \quad m,n \in \mathbb{N}$

$n^2 - 1 \leq s \qquad \boxed{n \leq X}$

$n^2 \leq s + 1$

$n \leq n^2 \ \forall n : n \leq s + 1 \qquad \exists n \in \mathbb{N} : n > s + 1$

d) $\{ 2^{-n} + 3^{-n} : n \in \mathbb{N} \}$

d) $\{2^{-n}+3^{-n} : n\in\mathbb{N}\}$

e) $\left\{(-1)^{-n} + \dfrac{1}{m^2+m-1} \,;\, n,m\in\mathbb{N}\right\}$

sup, inf, max, min

$n \leq 2^n \leq 3^n$