Cvičení 10.11.

úterý 10. listopadu 2020 13:42

F je primitivní fce k f (na I) pokud $F' = f$

$\cos x \to \sin x$ F, G p.f. k f

$(\sin x)' = \cos x$ pak $F - G$ je konstantní

$(\sin x + 7)' = \cos x$

$\int f \overset{c}{=} F$ (funkce) $\left(= F + C \quad C \in \mathbb{R}\right)$

↑ množina funkcí

$\int x^7 dx \overset{c}{=} \frac{x^8}{8}$

$(x^\alpha)' = \alpha x^{\alpha-1}$ (0 · $\frac{1}{x}$)

$\int x^\alpha dx \overset{c}{=} \frac{x^{\alpha+1}}{\alpha+1} \quad \alpha \neq -1$

$(e^x)' = e^x \quad \int \frac{1}{x} dx \overset{c}{=} \log|x| \quad (-\infty, 0) \quad (0, \infty)$

$(e^{-x})' = -e^{-x} \quad \int e^x dx \overset{c}{=} e^x \quad \mathbb{R}$

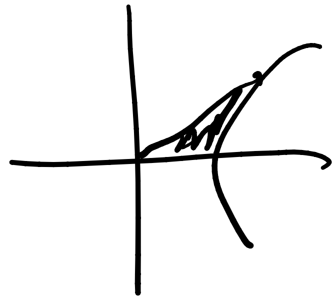
- $\int e^{-x} dx \overset{c}{=} -e^{-x}$

$\int \frac{1}{\sqrt{1-x^2}} dx \overset{c}{=} \arcsin x \quad | \quad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx \overset{c}{=} \arcsin x \qquad \Big| \qquad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$\overset{c}{=} -\arccos x \qquad (-1,1)$$

$$\int \frac{1}{1+x^2}\,dx \overset{c}{=} \operatorname{arctg} x \qquad \mathbb{R}$$



$$\int \frac{1}{\sqrt{x^2-1}}\,dx \overset{c}{=} \operatorname{argcosh}$$

~~arccosh~~ argcosh

acosh

$$\int \frac{1}{\sqrt{1+x^2}} \overset{c}{=} \operatorname{argsinh}$$

$$\int (f+g)' = \int f' + g'$$

F PF k f

G PF k g

$$f + g \overset{c}{=} \int f' + \int g'$$

potom

$\alpha F + \beta G$ je PF k $\alpha f + \beta g$

$$\int \sin x + \frac{x^3}{5} + \frac{1}{\cos^2 x}\,dx = -\int -\sin x\,dx + \frac{1}{5}\int x^3\,dx + \int \frac{1}{\cos^2 x}\,dx$$

$$\overset{c}{=} -\cos x + \frac{1}{5}\cdot\frac{x^4}{4} + \tan x$$

(——)(——)( ) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + k\pi \quad k \in \mathbb{Z}$

$$f\cdot g \overset{c}{=} \int (f\cdot g)' = \int f'g + \int f g' \qquad \int f g'$$

$f\cdot g \overset{c}{=} \int (f\cdot g)' = \int f'g + \int f g'$ $\qquad \int f g'$

$$\left[\int f g' = f\cdot g - \int f' g \right] \quad ?$$

Per partes

(1) $\int x e^x \, dx = f\cdot g - \int f' g = x e^x - \int 1\cdot e^x \overset{c}{=} x e^x - e^x$

$(x)' = 1$ $\quad f = x,\ g' = e^x \quad f' = 1,\ g = e^x \qquad f = e^x,\ g' = x \qquad f' = e^x,\ g = x^2 \quad \int x^2 e^x$

(1) $\int P(x) f(x)$

$P(x)$: polynom

(dobře se integrují) $e^x, e^{-x}, a^x, \sin x, \cos x, \sinh x, \cosh x$

$$\int x^3 \sin x = \left| \begin{matrix} f = x^3 & f' = 3x^2 \\ g' = \sin x & g = -\cos x \end{matrix} \right| = x^3\cdot(-\cos x) + \int 3x^2 \cos x \, dx$$

(x^3: $P(x)$, $\sin x$: $f(x)$)

$$= \left| \begin{matrix} f = 3x^2 & f' = 6x \\ g' = \cos x & g = \sin x \end{matrix} \right| = -x^3\cos x + 3x^2 \sin x - \int 6x \sin x \, dx$$

$$= \left| \begin{matrix} f = 6x & f' = 6 \\ g' = \sin x & g = -\cos x \end{matrix} \right| = -x^3\cos x + 3x^2 \sin x - \left(6x\,(-\cos x) - \int 6\,(-\cos x)\, dx\right)$$

během cvičení
zde bude opravené

$$= -x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x$$

během cvičení zde bylo opačné znaménko

(2) $\int P(x)\, f(x)$ (má příznivé derivace)

$P(x)$ … polynom

$f(x)$ … $\log x$, $\operatorname{arctan} x$, $\arccos x$, $\operatorname{argsinh} x$, …)

$\log^7 x$ $\lg^3 x$

$$\int x \operatorname{arctg} x = \frac{x^2}{2}\operatorname{arctg} x - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx$$

| $f = \operatorname{arctg} x$ | $f' = \frac{1}{1+x^2}$ |
| $g' = x$ | $g = \frac{x^2}{2}$ |

$$\int \frac{x^2+1}{1+x^2} - \frac{1}{1+x^2}\,dx$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx$$

$\int x = \frac{x^2}{2}$

$$= \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$

$$\overset{c}{=} \frac{x^2}{2}\operatorname{arctg} x - \frac{x}{2} + \frac{1}{2}\operatorname{arctg} x \qquad x \in \mathbb{R}.$$

$$\int 1\cdot \log x\,dx = x\log x - \int x\cdot\frac{1}{x}\,dx = x\log x - \int 1 \overset{c}{=} x\log x - x$$

| $f = \log x$ | $f' = \frac{1}{x}$ |
| $g' = 1$ | $g = x$ |

$$\int \log^2 x \, dx = x \log^2 x - \int 2 \cdot \log x \cdot \frac{1}{x} \cdot x$$

$f = \log^2 x \quad f' = 2 \cdot \log x \cdot \frac{1}{x}$

$g' = 1 \quad g = x$

$\int \operatorname{arctg} x \, dx \rightarrow$ substituce

(3) $\int f \cdot g \longrightarrow \int \sin^2 x \, dx \quad \int \cos^2 x \, dx$

$\uparrow$

$(e^x, e^{-x}, \sin x, \cos x, \sinh x, \cosh x) \quad \int \sin^2 x \, e^x$

$$\int \sin^2 x \, dx = -\sin x \cdot \cos x + \int \cos^2 x \, dx = -\sin x \cos x + \int 1 - \sin^2 x \, dx$$

$f = \sin x \quad f' = \cos x \qquad \sin^2 x + \cos^2 x = 1$

$g' = \sin x \quad g = -\cos x$

$$2\int \sin^2 x = -\sin x \cos x + \int 1 \, dx =$$

$$\int \sin^2 x \, dx = \frac{x - \sin x \cos x}{2} \qquad x \in \mathbb{R}$$

$$\int \sin x \, e^x \, dx = \left| \begin{matrix} f = e^x & f' = e^x \\ g' = \sin x & g = -\cos x \end{matrix} \right| = -e^x \cos x + \int \cos x \, e^x \, dx$$

$$= \left| \begin{matrix} f = e^x & f' = e^x \\ g' = \cos x & g = \sin x \end{matrix} \right| = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$= \left| \begin{matrix} f = e^x & f' = e^x \\ g' = \cos x & g = \sin x \end{matrix} \right| = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

$$\int \sin x \, e^x \, dx \overset{c}{=} e^x \frac{\sin x - \cos x}{2} \qquad x \in \mathbb{R}.$$

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(4) $\int \frac{1}{(1+x^2)^n} \, dx$ $\qquad \int 1 \cdot \frac{1}{1+x^2} \overset{PP}{\rightarrow} \int \frac{1}{(1+x^2)^2}$

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(1), (2) úpravy + tabulka

(12), (13) $\quad$ (11) $\rightarrow \int \sin^2 x$

(1) – (11)

$$\int \frac{x}{x} \cos(\log x) \, dx = \int \cos t \cdot e^t \cdot dt$$

$t = \log x \qquad x = e^t$

$dt = \frac{1}{x} dx$

$$\int \cos^2 x \cdot \cos^2 x \, dx = \int \cos x \cdot \cos^3 x$$

$$\int \sin^2 x + \int \cos^2 x = \int 1 = x$$

$$\int \cos^2 x = x - \int \sin^2 x$$

$$= x - \frac{x}{2} + \frac{\sin x \cos x}{2}$$

$$= \frac{x + \sin x \cos x}{2}$$

$= \left| f = \cos^3 x \quad f' = -3\cos^2 x \cdot \sin x \right.$

$1 - \cos^2 x$

$$= \left| \begin{array}{ll} f = \cos^3 x & f' = -3\cos^2 x \cdot \sin x \\ g' = \cos x & g = \sin x \end{array} \right.$$

$$1 - \cos^2 x$$

$$= \frac{x + \sin x \cos x}{2}$$

$$\int \cos^4 x \, dx = \cos^3 x \cdot \sin x + 3 \int \cos^2 x \cdot \sin^2 x$$

$$= \cos^3 x \sin x + 3 \int \cos^2 x - 3 \int \cos^4 x \, dx$$

$$\int 1 \cdot \sin(\log x) \, dx = x \sin(\log x) - \int \cos(\log x) \, dx$$

$$f = \sin(\log x) \quad f' = \frac{1}{x}(\cos(\log x))$$

$$g' = 1 \quad g = x$$