Řešení 2. testu

sobota 5. prosince 2020 20:47

(1) $f(x) = (1+\sin^7 x)^{|\sin x|} = e^{|\sin x| \cdot \log(1+\sin^7 x)}$

Protože $(|\sin x|)' = \operatorname{sgn}(\sin x) \cdot \cos x, \quad x \neq k\pi, k \in \mathbb{Z},$

$$(\log(1+\sin^7 x))' = \frac{7\sin^6 x \cos x}{1+\sin^7 x}, \quad x \in \mathbb{R},$$

dostáváme

$$f'(x) = (|\sin x| \cdot \log(1+\sin^7 x))' \cdot e^{|\sin x| \cdot \log(1+\sin^7 x)}$$

$$= \left[\operatorname{sgn}(\sin x) \cdot \cos x \cdot \log(1+\sin^7 x) + |\sin x| \cdot \frac{7\sin^6 x \cos x}{1+\sin^7 x}\right] (1+\sin^7 x)^{|\sin x|},$$

$x \neq k\pi, k \in \mathbb{Z}.$

f je spojitá na $\mathbb{R}$ (složení spojitých funkcí) a tedy

$$f'_\pm(k\pi) = \lim_{x \to k\pi\pm} f'(x) = (\pm 1 \cdot 1 \cdot 0 + 0 \cdot 0) \cdot 1^0 = 0$$

Tedy $f'(k\pi) = 0 \quad k \in \mathbb{Z}.$

(2)

$$I = \int \frac{\log x + 4}{x \log x (\log^2 x - 2)} dx = \left| \begin{array}{l} t = \log x \\ dt = \frac{1}{x} dx \end{array} \right| = \int \frac{t+4}{t(t^2-2)} dt$$

$$\frac{t+4}{t(t^2-2)} = \frac{t+4}{t(t-\sqrt{2})(t+\sqrt{2})} = \frac{A}{t} + \frac{B}{t-\sqrt{2}} + \frac{C}{t+\sqrt{2}}$$

$$A = \frac{0+4}{(0-\sqrt{2})(0+\sqrt{2})} = -2, \quad B = \frac{\sqrt{2}+4}{\sqrt{2}\cdot(\sqrt{2}+\sqrt{2})} = \frac{\sqrt{2}+4}{4}, \quad C = \frac{-\sqrt{2}+4}{-\sqrt{2}(-\sqrt{2}-\sqrt{2})}$$

Tedy

$$I = -2\int \frac{1}{t} dt + \frac{\sqrt{2}+4}{4} \int \frac{1}{t-\sqrt{2}} dt + \frac{4-\sqrt{2}}{4} \int \frac{1}{t+\sqrt{2}} dt$$

$$\overset{c}{=} -2 \log|t| + \frac{\sqrt{2}+4}{4} \log|t-\sqrt{2}| + \frac{4-\sqrt{2}}{4} \log|t+\sqrt{2}|$$

$$= -2 \log|\log x| + \frac{\sqrt{2}+4}{4} \log|\log x - \sqrt{2}| + \frac{4-\sqrt{2}}{4} \log|\log x + \sqrt{2}|$$

$D_{\log} = (0, \infty)$, $\log x = \pm\sqrt{2} \iff x = e^{\pm\sqrt{2}}$, $\log x = 0 \iff x = 1$

maximální intervaly tedy jsou:

$$(0, e^{-\sqrt{2}}), (e^{-\sqrt{2}}, 1), (1, e^{\sqrt{2}}), (e^{\sqrt{2}}, \infty).$$