(1) $$\lim_{x\to\infty} \frac{2x^2+1}{\sqrt{3x^4-6x^2+5}} = \lim_{x\to\infty} \frac{2+\frac{1}{x^2}}{\sqrt{3-\frac{6}{x^2}+\frac{5}{x^4}}} = \underline{\underline{\frac{2}{\sqrt{3}}}}$$

nejrychlejší člen jmenovatele

$\sqrt{\cdot}$ spojitá v 3

$a-b=\frac{a^2-b^2}{a+b}$

(2) $$\lim_{x\to\infty} x\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) = \lim_{x\to\infty} x\cdot\frac{x^2+1-(x^2-1)}{\sqrt{x^2+1}+\sqrt{x^2-1}}$$

$$=\lim_{x\to\infty}\frac{2x}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty}\frac{2}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}} = \frac{2}{1+1} = \underline{\underline{1}}$$

$\sqrt{\cdot}$ spojitá v 1

(3) $$\lim_{x\to\infty} x^{\frac{4}{3}}\left(\sqrt[3]{x^2+1}-\sqrt[3]{x^2-1}\right) = \lim_{x\to\infty} x^{\frac{4}{3}} \frac{\overbrace{x^2+1-(x^2-1)}^{\text{"}2\text{"}}}{(x^2+1)^{\frac{2}{3}}+((x^2+1)(x^2-1))^{\frac{1}{3}}+(x^2-1)^{\frac{2}{3}}}$$

$$=\lim_{x\to\infty}\frac{2}{\left(1+\frac{1}{x^2}\right)^{\frac{2}{3}}+\left(1+\frac{1}{x^2}\right)^{\frac{1}{3}}\left(1-\frac{1}{x^2}\right)^{\frac{1}{3}}+\left(1-\frac{1}{x^2}\right)^{\frac{2}{3}}} = \frac{2}{1+1+1} = \underline{\underline{\frac{2}{3}}}$$

(1) $$\lim_{x\to 0}\frac{\operatorname{tg} x - x}{x-\sin x} = \lim_{x\to 0}\frac{\frac{1}{\cos^2 x}-1}{1-\cos x} = \lim_{x\to 0}\frac{1}{\cos^2 x}\cdot\frac{1-\cos^2 x}{1-\cos x}$$

"$\frac{0}{0}$"

$$=\lim_{x\to 0}\frac{1}{\cos^2 x}\cdot\lim_{x\to 0}(1+\cos x) = 1\cdot 2 = \underline{\underline{2}}$$

(3) "$\frac{0}{0}$" $\to$ l'Hospital

$$\lim_{x\to 0}\frac{1-\cos(x^2)}{x^2\sin(x^2)} \overset{\text{"}\frac{0}{0}\text{" l'Hospital}}{=} \lim_{x\to 0}\frac{2x\sin(x^2)}{2x\sin(x^2)+2x^3\cos(x^2)} = \lim_{x\to 0}\frac{\sin(x^2)}{\sin(x^2)+x^2\cos(x^2)}$$

$$=\lim_{x\to 0}\frac{2x\cos(x^2)}{2x\cos(x^2)+2x\cos(x^2)+2x^3\sin(x^2)} = \lim_{x\to 0}\frac{\cos(x^2)}{2\cos(x^2)+2x^2\sin(x^2)} = \frac{1}{2+0} = \underline{\underline{\frac{1}{2}}}$$

(4)

$$\lim_{x\to\frac{\pi}{4}}(\mathrm{tg}\,x)^{\mathrm{tg}\,2x} = \lim_{x\to\frac{\pi}{4}} e^{\mathrm{tg}\,2x\cdot\log(\mathrm{tg}\,x)} = \underline{\underline{e^{-1}}}$$

e^x spojitá v -1

"$\frac{0}{0}$" $\to$ l'Hospital

$$\lim_{x\to\frac{\pi}{4}}\mathrm{tg}\,2x\cdot\log(\mathrm{tg}\,x) = \lim_{x\to\frac{\pi}{4}}\frac{\log(\mathrm{tg}\,x)}{\mathrm{cotg}\,2x} = \lim_{x\to\frac{\pi}{4}}\frac{\frac{1}{\cos^2 x}\cdot\frac{1}{\mathrm{tg}\,x}}{-\frac{2}{\sin^2 2x}}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{-\sin^2 2x}{2\cdot\sin x\cos x} = \frac{-1}{2\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}} = -1$$

(5) Pro $0<x\leq 1$ platí

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt[8]{x}} = \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{\sqrt[4]{x}}} = \sqrt{x^{\frac{3}{4}}+\sqrt{\frac{x+\sqrt{x}}{\sqrt{x}}}}$$

$$=\sqrt{x^{\frac{3}{4}}+\sqrt{\sqrt{x}+1}} \leq \sqrt{1+\sqrt{\sqrt{1}+1}}$$

(4)

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}} = \lim_{x\to\infty} \sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}$$

$$= \lim_{x\to\infty} \sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}} = \lim_{x\to\infty} \sqrt{1+\sqrt{\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{1}{x^3}}_{\to 0}}} = \sqrt{1+\sqrt{0+0}} = \underline{\underline{1}}$$

$\sqrt{\cdot}$ spojitá zprava v 0

$\sqrt{\cdot}$ spojitá v 1