úterý 10. listopadu 2020 18:52

(1)
$$\int \left(\frac{1-x}{x}\right)^2 dx = \int \frac{1-2x+x^2}{x^2}\,dx = \int \frac{1}{x^2} - \frac{2}{x} + 1\,dx$$
$$= \int \frac{1}{x^2}dx - 2\int \frac{1}{x}dx + \int 1\,dx \overset{c}{=} -\frac{1}{x} - 2\log|x| + x$$

$\boxed{\text{maximální intervaly } (-\infty,0),(0,\infty)}$

(2)
$$\int \tan^2 x\,dx = \int \frac{\sin^2 x}{\cos^2 x}dx = \int \frac{1-\cos^2 x}{\cos^2 x}dx$$
$$= \int \frac{1}{\cos^2 x}dx - \int 1\,dx \overset{c}{=} \boxed{\operatorname{tg} x - x}$$

$\boxed{\text{maximální intervaly } \left(-\frac{\pi}{2}+k\pi, \frac{\pi}{2}+k\pi\right) \quad k\in\mathbb{Z}}$

(4)
$$\int x^2 \sin x\,dx = \left|\begin{matrix} f=x^2 & f'=2x \\ g'=\sin x & g=-\cos x \end{matrix}\right| = -x^2\cos x + \int 2x\cos x$$
$$= \left|\begin{matrix} f=2x & f'=2 \\ g'=\cos x & g=\sin x \end{matrix}\right| = -x^2\cos x + 2x\sin x - 2\int \sin x$$
$$\overset{c}{=} \boxed{-x^2\cos x + 2x\sin x + 2\cos x} \qquad \boxed{\text{maximální interval } \mathbb{R}}$$

7)

$$\int \log^2 x\,dx = \left|\begin{array}{ll} f=\log^2 x & f'=\frac{2}{x}\cdot\log x \\ g'=1 & g=x \end{array}\right| = x\log^2 x - \int x\cdot\frac{2}{x}\cdot\log x\,dx$$

$$= x\log^2 x - 2\int \log x\,dx = \left|\begin{array}{ll} f=\log x & f'=\frac{1}{x} \\ g'=1 & g=x \end{array}\right|$$

$$= x\log^2 x - 2x\log x + 2\int 1\,dx \overset{C}{=} \boxed{x(\log^2 x - 2\log x + 2)}$$

maximální interval $(0,\infty)$

(8)

$$\int x \operatorname{argtanh} x\,dx = \left|\begin{array}{ll} f=\operatorname{arctanh} x & f'=\frac{1}{1-x^2} \\ g'=x & g=\frac{x^2}{2} \end{array}\right|$$

$$= \frac{x^2}{2}\operatorname{argtanh} x - \frac{1}{2}\int \frac{x^2}{1-x^2}$$

$$= \frac{x^2}{2}\operatorname{argtanh} x + \frac{1}{2}\int \frac{1-x^2}{1-x^2} - \frac{1}{1-x^2}\,dx$$

$$= \frac{x^2}{2}\operatorname{argtanh} + \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \frac{1}{1-x^2}\,dx$$

$$= \boxed{\frac{x^2}{2}\operatorname{argtanh} x + \frac{x}{2} - \frac{1}{2}\operatorname{argtanh} x}$$

maximální interval $(-1,1)$

(9) $$\int e^x \cos x\,dx = \left| \begin{matrix} f = \cos x & f' = -\sin x \\ g' = e^x & g = e^x \end{matrix} \right| = e^x \cos x + \int e^x \sin x\,dx$$

$$= \left| \begin{matrix} f = \sin x & f' = \cos x \\ g' = e^x & g = e^x \end{matrix} \right| = e^x \cos x + e^x \sin x - \int e^x \cos x\,dx$$

Tedy $$\int e^x \cos x\,dx = \boxed{\frac{e^x}{2}(\sin x + \cos x)}$$

maximální interval $\mathbb{R}$

(10) $$\int \cos^2 x\,dx = \left| \begin{matrix} f = \cos x & f' = -\sin x \\ g' = \cos x & g = \sin x \end{matrix} \right| = \sin x \cos x + \int \sin^2 x\,dx$$

$$= \sin x \cos x + \int 1 - \cos^2 x\,dx = \sin x \cos x + x - \int \cos^2 x\,dx$$

Tedy $$\int \cos^2 x\,dx = \boxed{\frac{\sin x \cos x + x}{2}}$$

maximální interval $\mathbb{R}$

(11)

$$\int \sin^4 x\,dx = \left| \begin{array}{ll} f = \sin^3 x & f' = 3\sin^2 x \cos x \\ g' = \sin x & g = -\cos x \end{array} \right|$$

$$= -\sin^3 x \cos x + 3\int \sin^2 x \cos^2 x\,dx$$

$$= -\sin^3 x \cos x + 3\int \sin^2 x (1-\sin^2 x)\,dx$$

$$= -\sin^3 x \cos x + 3\int \sin^2 x\,dx - 3\int \sin^4 x\,dx$$

Tedy

$$\int \sin^4 x\,dx = -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\int \sin^2 x\,dx$$

$$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\int 1-\cos^2 x\,dx$$

$$= -\frac{1}{4}\sin^3 x \cos x + \frac{3}{4}\left(x - \frac{\sin x \cos x + x}{2}\right)$$

$$= \boxed{-\frac{1}{4}\sin^3 x \cos x - \frac{3}{8}(\sin x \cos x - x)}$$

maximální interval $\mathbb{R}$

(12) $\int \sin(\log x)\,dx = \left| \begin{array}{ll} f = \sin(\log x) & f' = \frac{1}{x}\cos(\log x) \\ g' = 1 & g = x \end{array} \right|$

$= x\sin(\log x) - \int x \cdot \frac{1}{x}\cos(\log x)\,dx = x\sin(\log x) - \int \cos(\log x)\,dx$

$= \left| \begin{array}{ll} f = \cos(\log x) & f' = -\frac{1}{x}\sin(\log x) \\ g' = 1 & g = x \end{array} \right|$

$= x\sin(\log x) - x\cos(\log x) - \int \sin(\log x)\,dx$

Tedy $\int \sin(\log x)\,dx = \boxed{\frac{x}{2}\left(\sin(\log x) - \cos(\log x)\right)}$

$\boxed{\text{maximální interval} \quad (0,\infty)}$