úterý 27. října 2020 11:58

$$K=\{0\}\cup\{\tfrac{1}{n} : n\in\mathbb{N}\} \Rightarrow \dim_r(K)=\tfrac{1}{2}$$

$$\dim_r(\mathbb{Q}\cap[0,1])=1$$

$$\dim_r(A_1\cup A_2\cup\dots\cup A_k)=\max \dim_r(A_i)$$

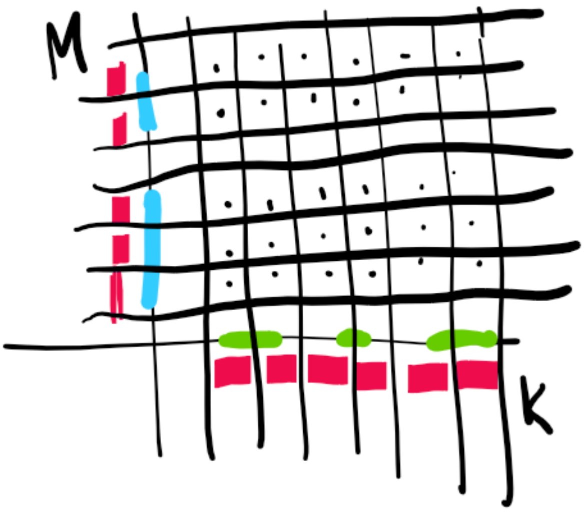
$K\subseteq\mathbb{R}^d$ $\quad M\subseteq\mathbb{R}^m$ nonempty bounded

then

$$\dim_r(K\times M)=?$$

$$\dim_B(K)=\dim_r(K)$$

$$\dim_B(M)=\dim_r(M)$$



$$B(K\times M,\varepsilon)=B(K,\varepsilon)\cdot B(M,\varepsilon)$$

$$B(K\times M,\varepsilon)=B(K,\varepsilon)\cdot B(M,\varepsilon)$$

$$\frac{\log(B(K\times M,\varepsilon))}{\log\frac{1}{\varepsilon}}=\frac{\log B(K,\varepsilon)+\log B(M,\varepsilon)}{\log\frac{1}{\varepsilon}}$$

limsup

$\varepsilon\to 0$

$$\limsup_{\varepsilon \to 0+} \quad \log \varepsilon \qquad \varepsilon_n \searrow 0$$

$$\overline{\dim}_\mu K \times M \geq \overline{\dim}_\mu K + \underline{\dim}_\mu M$$

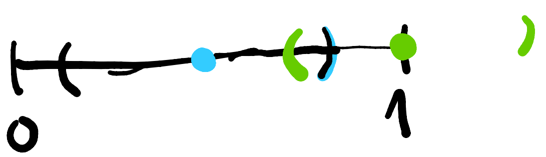
$$\underline{\dim}_\mu K \times M \leq \underline{\dim}_\mu K + \overline{\dim}_\mu M$$

- $\dim_\mu K \times M = \dim_\mu K + \dim_\mu M$

Ahlfors regular sets $\to$ s-regular sets $\quad s \geq 0$

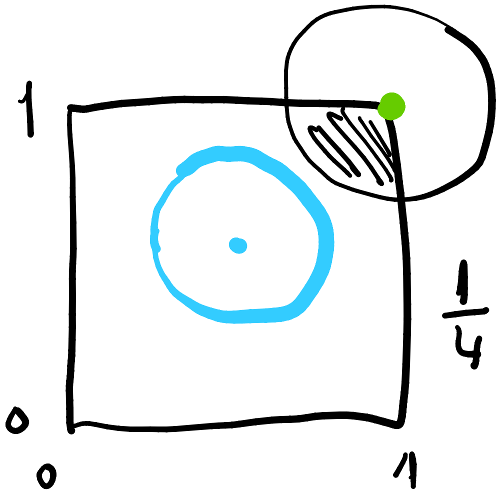
A set $K \subseteq \mathbb{R}^d$ is called s-regular if there are constants $\alpha, \beta, r_0 > 0$ and a Borel measure μ s.t.

- $0 < \mu(K) \leq \mu(\mathbb{R}^d) < \infty$
- $\alpha r^s \leq \mu(B(x,r)) \leq \beta r^s \quad x \in M, 0 < r \leq r_0$



$$\mu = \lambda_1|_{[0,1]} \qquad r_0 = \frac{1}{2}$$

$$r \leq \mu(B(x,r)) \leq 2r \qquad \alpha = 1 \quad \beta = 2 \quad s = 1$$



$\mu = \lambda_2|_{[0,1]^2}$ $\quad r = \frac{1}{2}$

$\frac{1}{4}\pi r^2 \le \mu(B(x,r)) \le \pi r^2$

$\alpha = \frac{\pi}{4} \quad \beta = \pi \quad s = 2$

s-sets

• $P(\{0\}) = \frac{1}{2} = P(\{1\})$

γ on subsets of $\{0,1\}^{\mathbb{N}}$

$\gamma(\{1,\ldots\}) = \frac{1}{2}$

$\gamma(\{1,0,\ldots\}) = \frac{1}{4}$

$\{1,0,0,\ldots\}$

$\omega \in \{0,1\}^{\mathbb{N}}$

$\varphi \mapsto x \in C_{\frac{1}{3}}$

$A \subseteq C_{\frac{1}{3}}$

$\bar{\mu}(A) = \varphi_* \gamma$

$C_{\frac{1}{3}}$ is $\frac{\log 2}{\log 3}$-set

$A \subseteq \mathbb{R}$

• $\mu(A) = \bar{\mu}(A \cap C_{\frac{1}{3}})$

L: If K is nonempty bounded s-set.

Then $\underline{\dim}_M K = \overline{\dim}_M K = s$.

P: $C(K,\varepsilon) \longrightarrow K \subseteq \bigcup B(x_i,\varepsilon)$ ↙ optimal covering

$$0 < \mu(K) \le \mu\Big(\bigcup B(x_i,\varepsilon)\Big) \le \sum \mu\big(B(x_i,\varepsilon)\big)$$

$$\boxed{C(K,\varepsilon) \ge \underbrace{\frac{\mu(K)}{\beta}}_{B :=} \cdot \varepsilon^{-s}} \longleftarrow \mu(K) \le \beta \cdot \varepsilon^{s} \cdot C(K,\varepsilon)$$

$$\frac{\log C(K,\varepsilon)}{\log \frac{1}{\varepsilon}} \ge \frac{\log B + \log \overset{\left(\frac{1}{\varepsilon}\right)^s}{\varepsilon^{-s}}}{\log \frac{1}{\varepsilon}} = \underbrace{\frac{\log B}{\log \frac{1}{\varepsilon}}}_{\to 0} + \frac{s \cancel{\log \frac{1}{\varepsilon}}}{\cancel{\log \frac{1}{\varepsilon}}} \to s$$

$s \le \underline{\dim}_M K$.

<u>HW:</u> Prove: $S \geq \overline{\dim}_M K$

<u>HW:</u>

Prove that $C_{\frac{1}{3}}$ is $\frac{\log 2}{\log 3}$-set

(using measure μ above)