Fractals 15.12.

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$$K = \bigcup \varphi_i(K), \quad \mu = \sum r_i^D (\varphi_i)_* \mu$$

- $\operatorname{supp} \mu = K$

$$\underline{\Sigma_N} \quad \pi : \Sigma_N \to \mathbb{R}^d \qquad (1,3,2,\dots) \downarrow \omega \to x_\omega$$

$$\pi : \Sigma_N \xrightarrow{\text{onto}} K \qquad \varphi_{\omega|_n}(K) = K_{\omega|_n} \xrightarrow{n \to \infty} \{x_\omega\}$$

$$\gamma([I]) = r_I^D \qquad \bullet \to r_{(1)}^D$$

$\pi_* \gamma$ – measure on K

$$\nu = \pi_* \gamma = \mu \qquad \operatorname{supp} \nu = K$$

$$\sigma_i : \Sigma_N \to \Sigma_N \qquad \omega \to i\omega$$

- $\left[\varphi_i \circ \pi = \pi \circ \sigma_i \right]$ $\quad \omega \to i\omega \to x_{i\omega}$

$$\omega \to x_\omega \to \varphi_i(x_\omega) = x_{i\omega}$$

$$\nu = \sum r_i^D (\varphi_i)_* \nu \to \nu = \mu$$

$$\underline{\nu = \sum r_i^{\nu} (\varphi_i)_* \nu} \longrightarrow \underline{\nu = \mu}$$

- $$\boxed{\mathcal{J} = \sum r_i^D (\sigma_i)_* \mathcal{J}}$$

<u>Hutchinson theorem</u>

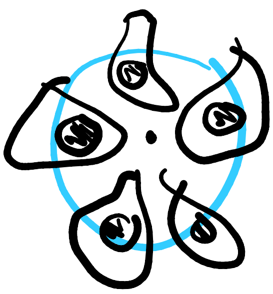
If $\varphi_1, \dots, \varphi_N$ satisfy OSC, then
K is D-regular $\quad (\sum r_i^D = 1)$

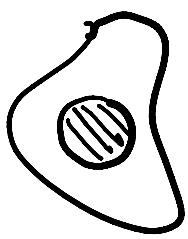
$\overline{U} \supseteq K$, U exists bounded

$\downarrow$

$\underline{\overline{U_I}} \supseteq K_I \qquad \underline{\mu_I} = \underline{\operatorname{supp} K_I}$

$U_I \cap U_J = \emptyset$ if I, J incomparable





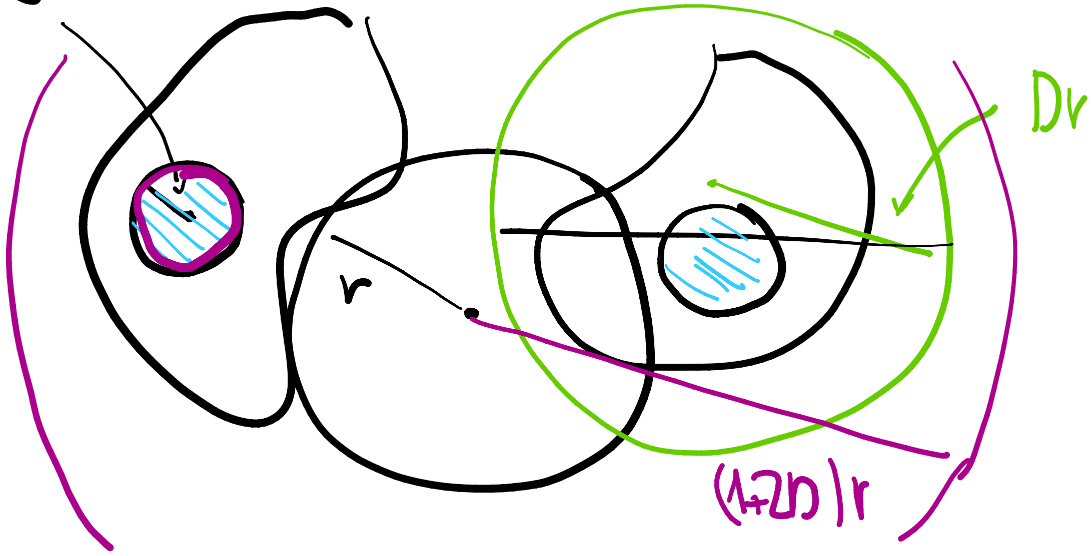
L: Let $\{U_i\}$ by a system of open sets
such that
- $U_i \cap U_j = \emptyset \quad i \neq j$

$r > 0$
- each U_i contains a ball of radius cr

$r>0$

- each U_i contains a ball of radius C ...
- is contained in a ball of radius Dr

Then $B(0,r)$ intersects at most $\dfrac{(1+2D)^d}{C^d}$ sets $\overline{U}_i$



$$\overline{U}_i \cap B(0,r) \Rightarrow \overline{U}_i \subseteq B(0,(1+2D)r)$$

$$N = \#\{i : \overline{U}_i \cap B(0,r) \neq \emptyset\}$$

$$N \cdot \cancel{V_d} \cdot (C \cdot \cancel{r})^d \leq \cancel{V_d} \cdot ((1+2D)\cancel{r})^d$$

$$\boxed{N \leq \frac{(1+2D)^d}{C^d}}$$

$\alpha, \beta > 0$

$$\alpha \cdot r^D \leq \mu(B(x,r)) \leq \beta r^D \qquad x \in K$$

$$0 < r \leq r_0$$

$0 < r \leq r_0$

$x \in K \longrightarrow x = x_\omega$

$\operatorname{diam}(K_I) = r_I \cdot \operatorname{diam} K$

$K_{\omega|n}$

$r_\emptyset \neq r_0$

$n \geq 1 \longleftarrow r < 1 \operatorname{diam} K$

$= K_\emptyset$

consider the smallest n s.t.

- $\boxed{\operatorname{diam} K \cdot r_{\omega|n} \leq r} \longrightarrow \boxed{K_{\omega|n} \subseteq B(x,r)}$

$\operatorname{diam} K \cdot r_{\omega|n-1} > r$ $\qquad r_{min} = \min r_i > 0$

- $\boxed{\operatorname{diam} K \, r_{\omega|n} > r \cdot r_{min}}$ $\qquad \boxed{\mu = \sum_{|I|=n} \mu_I}$

$$\mu(B(x,r)) \geq \mu_{\omega|n}(B(x,r)) \geq \mu_{\omega|n}(K_{\omega|n}) = r_{\omega|n}^D$$

$$> \frac{r^D \cdot r_{min}^D}{(\operatorname{diam} K)^d} = \alpha \cdot r^D \qquad \alpha = \frac{r_{min}^D}{(\operatorname{diam} K)^1} > 0$$

$$r > 0 \longrightarrow \overset{K}{\overset{\cup}{z}} = z_\omega$$

$$r_{min}\, r < r_{\omega|n} \leq r \qquad \omega \to \text{unique } n \text{ s.t.}$$

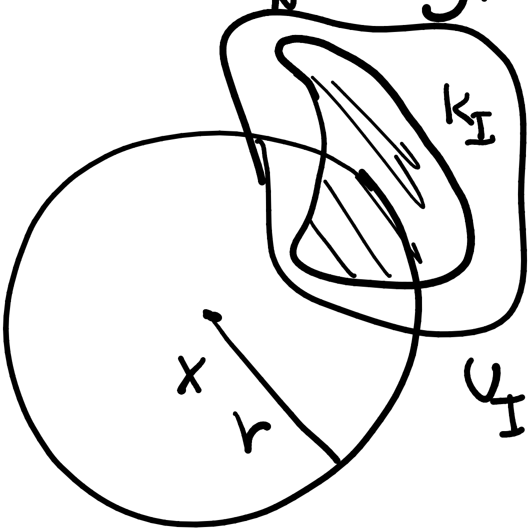
$r_{\min} r < r_{\omega|_n} \leq r$ $\qquad \omega \to$ unique n s.t.

$$\mathcal{I} = \{\omega|_n : r_{\min} r < r_{\omega|_n} \leq r\} \qquad r<1$$

- $K = \bigcup_{I\in\mathcal{I}} K_I \Rightarrow \mu = \sum_{I\in\mathcal{I}} \mu_I$ * HW1

- $I,J \in \mathcal{I} \Rightarrow I,J$ incomparable

$\omega \in \Sigma_N \Rightarrow \exists n : \omega_n \in \mathcal{I}$



U - bounded sensible open set
there is a ball of radius $\rho > 0$
and is contained is some ball
of radius $R > 0$

Then U_I contains a ball of radius $\boxed{r_I \rho}$
is contained in a ball of radius $\boxed{r_I R}$

$$I \in \mathcal{I} \Rightarrow r_{\min} r < r_I \leq r$$

$I \in \mathcal{I} \Rightarrow U_I$ contains ball of rad. $r_{\min} \cdot r \cdot \rho$

$I \in \mathcal{I} \Rightarrow U_I$ contains ball of rad. $r_{min} \cdot r \cdot \rho$

is contained —ıı— $r \cdot R$

$$J = \{I : K_I \cap B(x,r)\} \leq \frac{(1+2R)^d}{(r_{min}\cdot\rho)^d} = E$$

$$\Downarrow$$

$$\underbrace{\overline{U}_I \cap B(x,r)}$$

$\uparrow$

$L: C = r_{min} \cdot \rho$

$D = R$

$*+$

K_J

$$\boxed{\mu(B(x,r))} = \sum_{I \in J} \mu_I(B(x,r)) \leq \sum_{I \in J} \mu_I(\mathbb{R}^d) =$$

$$= \sum_{I \in J} r_I^D \leq \sum_{I \in J} r^D = \boxed{E r^D}$$

$\beta = E$ ▨

$\underline{\underline{r_I \ll r_J}}$ $\quad \underline{\underline{|J| \sim |I|}}$

If the system $(\varphi_1, \dots, \varphi_N)$ satisfies (OSC), then $\dim_H K = \underline{\dim}_M K = \overline{\dim}_M K = D$.

If K is D-regular $\to \boxed{0 < \mathcal{H}^D(K) < \infty}$

$$\alpha r^D < B(x,r) < \beta r^D$$

$$\alpha < \frac{B(x,r)}{r^D} < \beta$$

$$\uparrow$$

$$\limsup \frac{B(x,r)}{r^D}$$

$$\boxed{\mathcal{H}^D(\varphi_i(A)) = r_i^D \mathcal{H}^D(A)}$$

$$0 < \mathcal{H}^D(K) < \infty$$

$$\underline{\mathcal{H}^D(K)} = \mathcal{H}^D\left(\bigcup \varphi_i(K)\right) \overset{=}{\leq} \sum \mathcal{H}^D(\varphi_i(K))$$

$$= \sum r_i^D \mathcal{H}^D(K) = \underline{\mathcal{H}^D(K)}$$

So $\mathcal{H}^D(K_i \cap K_j) = 0$

Then $\frac{1}{\mathcal{H}^D(K)} \mathcal{H}^D\big|_K = \mu$ HW 2

$$\uparrow$$

$$\Lambda$$

$$\Lambda = \sum r_i^D (\varphi_i)_* \Lambda$$

$$\lambda = 2 r_i''(\varphi_i)_x \lambda$$