Fractals 10.11.

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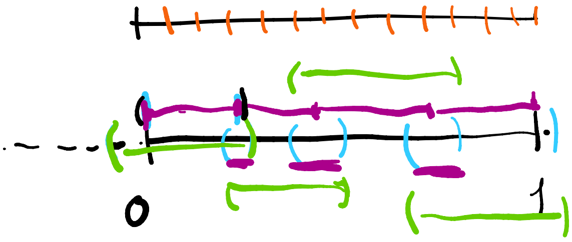


$\dim_{\mathcal{H}}([0,1]) = 1$

$$\mathcal{H}^{s(k)}_{\delta} = \inf\left\{ \sum \operatorname{diam}^s K_i : \bigcup K_i \supseteq K,\ \operatorname{diam} K_i \le \delta \right\}$$

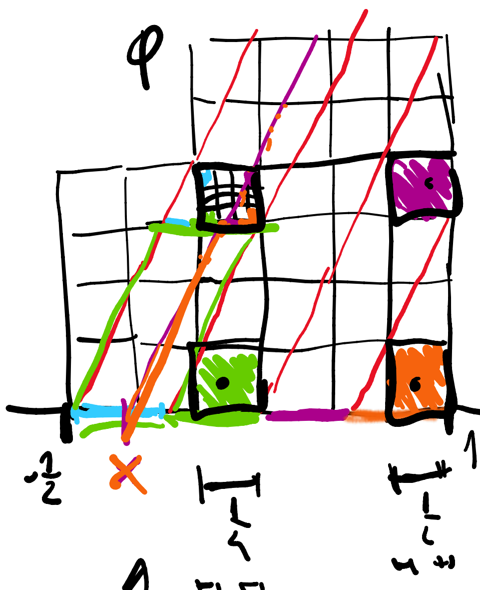
K_i are open convex

$s=1$

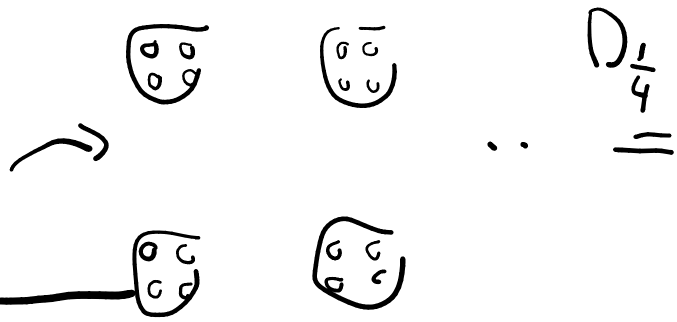
$$\varepsilon + \mathcal{H}^1_\delta([0,1]) \ge \mathcal{H}^1([0,1])$$

$$1 \ge \mathcal{H}^1_\delta([0,1]) \ge 1 \qquad \mathcal{H}^1_\delta([0,1]) = 1 \Rightarrow \mathcal{H}^1([0,1]) = 1$$

$$0 < \mathcal{H}^s(K) < \infty \Rightarrow \dim_{\mathcal{H}}(K) = s$$



$\varphi(D_{\frac{1}{4}}) = [-\frac{1}{2}, 1]$



$$\boxed{4 \cdot \left(\tfrac{1}{4}\right)^s = 1}$$

$\dim_{\mathcal{H}} C_{\frac{1}{4}} = \frac{1}{2}$;

$\mathcal{H}^s(\varphi(K)) = r^s \cdot \mathcal{H}^s(K)$

similarity

$\dim_{\mathcal{H}}(D_{\frac{1}{4}}) \ge \dim_{\mathcal{H}}([-\frac{1}{2},1]) = 1$

$|\varphi(x) - \varphi(y)| \le r \cdot |x-y|$

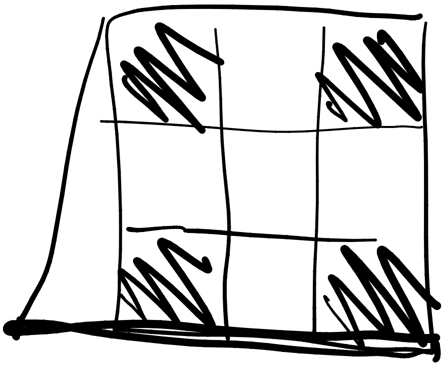
Lipschitz

$$\boxed{\text{HW: } \dim_{\mathcal{H}} K \le \underline{\dim}_B K}$$

$\mathcal{H}^s(\varphi(K)) \le r^s \mathcal{H}^s(K)$

$$\mathcal{H}^s(\varphi(K)) \le r^s \mathcal{H}^s(K)$$

$$\boxed{\dim_{\mathcal{H}}(\varphi(K)) \le \dim_{\mathcal{H}}(K)}$$



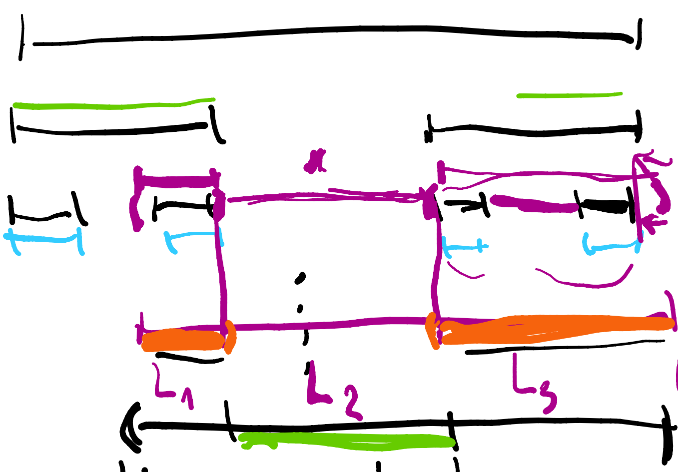
$$2\cdot\frac{\log 2}{\log 3} = \frac{\log 4}{\log 3} > 1$$
$$\dim_{\mathcal{H}}(C_{\frac13}) = 2\cdot\frac{\log 2}{\log 3} \ge 1$$

$C_{\frac13}$

$$C_{\frac13} \subseteq \bigcup U_i$$

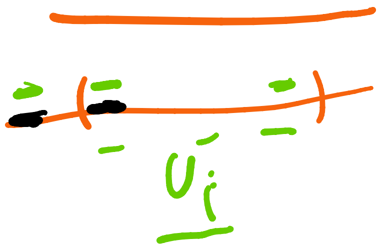
open convex

$$C_{\frac13} \subseteq \bigcup_{i=1}^{N} U_i$$



U_i open intervals

$$\boxed{L_2 \ge L_3, L_1} \to L_2 \ge \frac{L_3+L_1}{2}$$



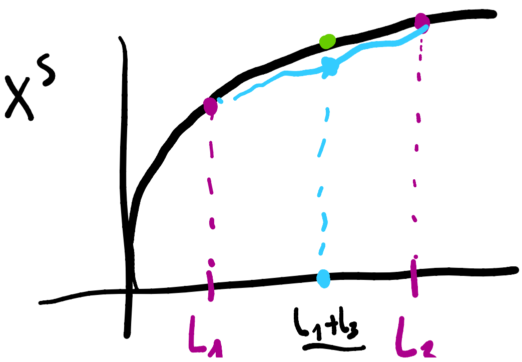
$$\operatorname{diam}^s U_i \ge L_1^s + L_3^s$$

$$(L_1+L_2+L_3)^s \ge \left(L_1 + \frac{L_3+L_1}{2} + L_3\right)^s = \left(\frac{3}{2}(L_1+L_3)\right)^s$$

$$= 2\left(\frac{L_1+L_3}{2}\right)^s \ge L_1^s + L_3^s$$

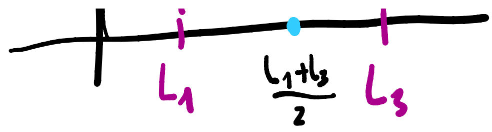
$0<s<1$

$$3^s = 2$$
$$3^{\frac{\log 2}{\log 3}} = 2$$



$$\log\left(3^{\frac{\log 2}{\log 3}}\right) = \log 2$$

$$\frac{\log 2}{\log 3} \cdot \log 3 = \log 2$$



$$\left(\frac{L_1+L_3}{2}\right)^s \geq \frac{L_1^s+L_3^s}{2}$$

$\mathcal{I}_k$ kth generation of base intervals

$$I \in \mathcal{I}_k \Rightarrow \operatorname{diam} I = \frac{1}{3^k} \qquad |\mathcal{I}_k| = 2^k$$

$$\sum_{I \in \mathcal{I}_k} \operatorname{diam}^s I = 2^k \cdot \left(\frac{1}{3^k}\right)^s = 2^k \cdot \frac{1}{2^k} = 1$$

$$1 \leq \mathcal{H}^s_\delta(C_{\frac{1}{3}}) \leq 1$$

$$\mathcal{H}^s(C_{\frac{1}{3}}) = 1 \Rightarrow \dim_{\mathcal{H}}(C_{\frac{1}{3}}) = s = \frac{\log 2}{\log 3}$$

$$A, B \subseteq [0,1]$$

$$\dim_{\mathcal{H}} A = \dim_{\mathcal{H}} B = 0$$

$$\dim_{\mathcal{H}}(A \times B) = 1$$

$m_1 \leq 0 < m$ - - - - - - - $\leq 0 \leq m_4$

m_1 —— m_2 $\quad m_3 \quad \ldots \quad m_4 \rightarrow$

$\leq 0 \leq$

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HW:

$\varphi: \mathbb{R}^2 \to \mathbb{R}$ $\quad \varphi: (x,y) \to x+y$

$\varphi(A \times B) \supseteq [0,1]$