Řešení DU 30.11.

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(1) $$I := \int \frac{x^3+5x^2-4x+4}{x^4-x^3-x+1}\,dx = \int \frac{x^3+5x^2-4x+4}{(x-1)^2(x^2+x+1)}\,dx$$

Platí
$$\frac{x^3+5x^2-4x+4}{(x-1)^2(x^2+x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+x+1} \qquad (*)$$

Zakrývacím pravidlem dostaneme
$$B = \frac{1+5-4+4}{1+1+1} = 2$$

Přenásobením rovnosti $(*)$ výrazem $(x-1)^2(x^2+x+1)$ dostaneme

$$\begin{aligned} x^3+5x^2-4x+4 &= A(x-1)(x^2+x+1) + B(x^2+x+1) + (Cx+D)(x-1)^2 \\ &= A(x^3-1) + B(x^2+x+1) + (Cx+D)(x^2-2x+1) \\ &= x^3(A+C) + x^2(B-2C+D) + x(B+C-2D) + (-A+B+D) \end{aligned}$$

Tedy porovnáním koeficientů u stejných mocnin dostaneme

$$\begin{aligned} 1 &= A+C \\ 5 &= B-2C+D \\ -4 &= B+C-2D \\ 4 &= -A+B+D \end{aligned} \quad \xrightarrow{B=2} \quad \begin{aligned} 1 &= A+C \;\to\; A=1 \\ 3 &= -2C+D \\ -6 &= C-2D \end{aligned} \Bigg\} \to \begin{aligned} D&=3 \\ C&=0 \end{aligned}$$

Tedy
$$I = \int \frac{1}{x-1}\,dx + \int \frac{2}{(x-1)^2}\,dx + \int \frac{3}{x^2+x+1}\,dx$$

Platí $\int \frac{1}{x-1}\,dx \overset{c}{=} \log|x-1|$, $\int \frac{2}{(x-1)^2}\,dx \overset{c}{=} \frac{-2}{x-1}$

A $\int \frac{3}{x^2+x+1}\,dx = 3\int \frac{1}{(x+\frac{1}{2})^2+\frac{3}{4}}\,dx = 4\int \frac{1}{\left(\frac{2x+1}{\sqrt{3}}\right)^2+1}\,dx$

$\overset{c}{=} 2\sqrt{3}\,\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)$

Celkově tedy

$$I \overset{c}{=} \log|x-1| - \frac{2}{x-1} + 2\sqrt{3}\,\operatorname{arctg}\left(\frac{2x+1}{\sqrt{3}}\right)$$

maximální intervaly $(-\infty,1),(1,\infty)$

(2)

$$\int \sin x \cdot \log^3(\cos x)\,dx = \begin{vmatrix} t = \cos x \\ dt = -\sin x\,dx \end{vmatrix} = -\int \log^3 t\,dt$$

$$= \begin{vmatrix} f = \log^3 t & f' = \frac{3}{t}\log^2 t \\ g' = 1 & g = t \end{vmatrix} = -t \cdot \log^3 t + 3\int \log^2 t\,dt$$

$$= \begin{vmatrix} f = \log^2 t & f' = \frac{2}{t}\log t \\ g' = 1 & g = t \end{vmatrix} = -t \cdot \log^3 t + 3t \cdot \log^2 t - 6\int \log t\,dt$$

$$= \begin{vmatrix} f = \log t & f' = \frac{1}{t} \\ g' = 1 & g = t \end{vmatrix} = -t\log^3 t + 3t \cdot \log^2 t - 6t \cdot \log t + 6\int 1\,dt$$

$$\overset{c}{=} t\left(-\log^3 t + 3\log^2 t - 6\log t + 6\right)$$

$$= \cos x\left(-\log^3(\cos x) + 3\log^2(\cos x) - 6\log(\cos x) + 6\right)$$

Maximální intervaly $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) + 2k\pi$, $k \in \mathbb{Z}$.

potřebujeme $\cos x > 0$