Řešení DÚ 2.11.

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(1)

$$\lim_{x\to 0} \frac{1+\sin x-\cos x}{1-\sin x-\cos x} = \lim_{x\to 0} \frac{1+\sin x-\cos x}{1-(\sin x+\cos x)} \cdot \frac{1+\sin x+\cos x}{1+\sin x+\cos x}$$

$$= \lim_{x\to 0} \frac{(1+\sin x)^2-\cos^2 x}{1-(\sin x+\cos x)^2}$$

$$= \lim_{x\to 0} \frac{1+2\sin x+\sin^2 x-\cos^2 x}{1-\sin^2 x-2\sin x\cos x-\cos^2 x}$$

$$= \lim_{x\to 0} \frac{2\sin x+2\sin^2 x}{-2\sin x\cos x}$$

$$= \lim_{x\to 0} \frac{1+\sin x}{-\cos x} = \underline{\underline{-1}}$$

(2) $\lim\limits_{x\to 1}\left(1+\sin(\pi x)\right)^{\cotg(\pi x)} = \lim\limits_{x\to 1} e^{\cotg(\pi x)\log(1+\sin \pi x)}$

$$= \lim_{x\to 1} e^{\cos(\pi x)\cdot\frac{\log(1+\sin(\pi x))}{\sin(\pi x)}}$$

$$= e^{(-1)\cdot 1} = \frac{1}{e}$$

Protože • $\lim\limits_{x\to 1}\dfrac{\log(1+\sin(\pi x))}{\sin(\pi x)} = 1$ podle VOLSF

$f(x) = \sin(\pi x)$ $\quad g(x) = \dfrac{\log(1+x)}{x}$ $\quad g\circ f(x) = \dfrac{\log(1+\sin(\pi x))}{\sin(\pi x)}$

$\lim\limits_{x\to 1} f(x) = 0 \qquad \lim\limits_{x\to 0} g(x) = 1 \qquad x\in P(1,1) \Rightarrow f(x)\neq 0$

A B C A (P) B

A dále • $\cos(\pi x)$ je spojitý v 1, $\cos(\pi) = -1$

• e^x je spojitá v -1