Řešení DÚ 23.1

sobota 21. listopadu 2020 5:11

(1)

$$\int (x^2+2x)7^x\,dx = \left| \begin{matrix} f = x^2+2x & f' = 2x+2 \\ g' = 7^x & g = \frac{7^x}{\log 7} \end{matrix} \right| = (x^2+2x)\frac{7^x}{\log 7} - \frac{2}{\log 7}\cdot\int (x+1)7^x\,dx$$

$$= \left| \begin{matrix} f = x+1 & f' = 1 \\ g' = 7^x & g = \frac{7^x}{\log 7} \end{matrix} \right| = (x^2+2x)\frac{7^x}{\log 7} - \frac{2}{\log 7}\left((x+1)\frac{7^x}{\log 7} - \frac{1}{\log 7}\int 7^x\,dx\right)$$

$$\overset{c}{=} \underline{\underline{\frac{7^x}{\log 7}(x^2+2x) - \frac{2\cdot 7^x}{\log^2 7}(x+1) + \frac{2\cdot 7^x}{\log^3 7}}} \qquad x \in \mathbb{R}.$$

(2)

$$\int \sin x \sinh x\,dx = \left| \begin{matrix} f = \sin x & f' = \cos x \\ g' = \sinh x & g = \cosh x \end{matrix} \right| = \sin x\cosh x - \int \cos x \cosh x\,dx$$

$$= \left| \begin{matrix} f = \cos x & f' = -\sin x \\ g' = \cosh x & g = \sinh x \end{matrix} \right| = \sin x\cosh x - \left(\cos x\sinh x + \int \sin x \sinh x\,dx\right)$$

$$= \sin x\cosh x - \cos x \sinh x - \int \sin x\sinh x\,dx$$

Tedy

$$\int \sin x\sinh x\,dx \overset{c}{=} \underline{\underline{\frac{1}{2}\left(\sin x\cosh x - \cos x\sinh x\right)}} \qquad x \in \mathbb{R}.$$

(3)

$$\int \log(x^2+1)\,dx = \left| \begin{array}{ll} f = \log(x^2+1) & f' = \dfrac{2x}{x^2+1} \\ g' = 1 & g = x \end{array} \right|$$

$$= x\log(x^2+1) - 2\int \frac{x^2 \overset{+1-1}{}}{x^2+1}\,dx = x\log(x^2+1) - 2\int 1\,dx + 2\int \frac{1}{x^2+1}\,dx$$

$$\stackrel{C}{=} \underline{\underline{x\log(x^2+1) - 2x + 2\operatorname{arctg} x}} \qquad x \in \mathbb{R}$$