

Numerical software 2

ANGENER

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Lecture 2

Code ANGENER

It modifies the input mesh \mathcal{T}_h and creates $\mathcal{T}_h^{\text{NEW}}$

Two versions

`msekce.karlin.mff.cuni.cz/~dolejsi/angen/angen3.0.htm`
`msekce.karlin.mff.cuni.cz/~dolejsi/angen/angen3.1.htm`

Downloading

```
gunzip angener3.1.tar.gz  
tar xf angener3.1.tar
```

Instalation

```
make
```

Running (after preparation of data files)

```
./side
```

Files (1)

main.f	source file
solver.f	source file
rest.f	source file
moving.f	source file
swapping.f	source file
insert.f	source file
remove.f	source file
rembou.f	source file
work.inc	included file of array declaration
makefile	makefile used for translation

- work.inc declare the amount of allocated memory
- Error in function MEMO: 10200 values miss in blank common
 ⇒ not enough of memory
- increase dimension of array **work** in **work.inc**
- rm -f main.o & make

Files (2)

paramet	sample of input file
profiles.01	sample of input file – $\Omega = [0, 1] \times [0, 1]$
triang.01	sample of input file – $\Omega = [0, 1] \times [0, 1]$
profiles.gam	sample of input file – GAMM channel
triang.gam	sample of input file – GAMM channel
profiles.2na	sample of input file – double NACA profile
triang.2na	sample of input file – double NACA profile
profiles.dca	sample of input file – periodic DCA08 profile
triang.dca	sample of input file – periodic DCA08 profile
spline.f	first source code for spline interpolation
library.f	library subroutine for spline code
readme	first instruction
manual.ps	description of ANGENER code

Basic description (1)

Modes of ANGENER

- uniform mesh
- adapted mesh

Uniform mesh

Input files:

- `paramet` - contains the parameters for AMA algorithm,
- `profiles` - contains the description of curved parts of $\partial\Omega$,
- `triang` - input triangulation \mathcal{T}_h ,

Output files:

- `triangx` - new triangulation $\mathcal{T}_h^{\text{NEW}}$,
- `mesh` - figure of $\mathcal{T}_h^{\text{NEW}}$ for direct use in gnuplot.

Adapted mesh

Input files:

- `paramet` - contains the parameters for AMA algorithm,
- `profiles` - contains the description of curved parts of $\partial\Omega$,
- `triang` - input triangulation \mathcal{T}_h ,
- `results` - results u_h computed on \mathcal{T}_h .

Output files:

- `triangx` - new triangulation $\mathcal{T}_h^{\text{NEW}}$,
- `resultsx` - results u_h interpolated on $\mathcal{T}_h^{\text{NEW}}$,
- `mesh` - figure of $\mathcal{T}_h^{\text{NEW}}$ for direct use in gnuplot.

Important variables and arrays in ANGENER

$nelem$		# of triangles of \mathcal{T}_h
$npoin$		# of nodes (vertices) of \mathcal{T}_h
$nbelm$		# of boundary segments of \mathcal{T}_h
nbc		# of boundary components
nbp		# of non-polygonal parts of $\partial\Omega$
$ndim$		# of components of solution (=1 for scalar equation)
$x(i)$	$i = 1, \dots, npoin$	x -coordinates of nodes of \mathcal{T}_h
$y(i)$	$i = 1, \dots, npoin$	y -coordinates of nodes of \mathcal{T}_h
$Ind(i,j)$	$i = 1, \dots, nelem, j = 1, 2, 3$	nodes of triangles
$lbn(i,j)$	$i = 1, \dots, nbelm, j = 1, 2$	nodes of boundary edge
$ibc(i)$	$i = 1, \dots, nbelm$	type of BC for boundary edges

$ityp$	type of construction, $0 \leq ityp \leq ndim$
$ndim$	number of component of seeking solution
ifv	type of mesh association
pos	positivity
$numel$	prescribed number of elements for uniform triangulation
ϵ_1	ϵ_1
p	p

- $ityp = 0$: a uniform triangulation is constructed
- $ityp > 0$: an adapted triangulation is constructed using $ityp - \text{th}$ component of u (it has $ndim$ components)
- $ifv = 0$: a cell-vertex scheme (continuous P_1)
- $ifv = 1$: a cell-centered scheme (discontinuous P_0)
- $pos > 0$: control the shape regularity

File paramet – example

```
0      ityp ( 0 - uniform trian, >0 - component of solution)
1      ndim
0      ifv ( 0 -cell vertex, 1- cell centered)
0.02    pos (=positivity)
500    numel (= c)
1.E+10  epsilon1
1.      p
```

File profile

a dense list of nodes defining nonpolygonal boundary:

2

16085

1. 0.249999985

0.999875009 0.249982268

0.999750018 0.249964535

:

0.999750018 0.250035465

0.999875009 0.250017732

1. 0.25

801

0.000 1.25

-0.00490878057 1.24999034

-0.00981743075 1.2499615

:

-0.0147257112 -1.24991322

-0.00981721189 -1.2499615

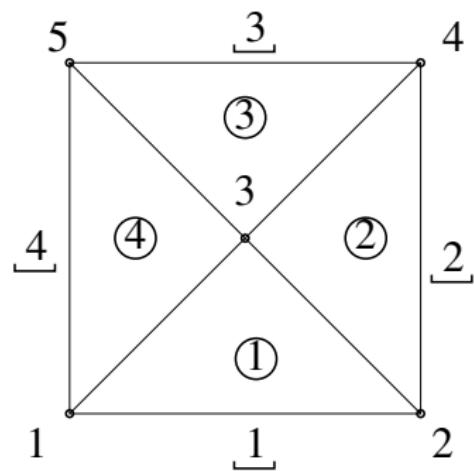
-0.00490856264 -1.24999034

0.000 -1.25

Files triang and triangx: unit square

5 4 4	npoint nelem nbelm
0.0 0.0 0 0	periodic boundary
0.0 0.0	x(1) y(1)
1.0 0.0	
0.5 0.5	
1.0 1.0	
0.0 1.0	x(5) y(5)
1 2 3	Ind(1,1) Ind(1,2) Ind(1,3)
2 4 3	
4 5 3	
5 1 3	Ind(4,1) Ind(4,2) Ind(4,3)
1 2 1	Ibn(1,1) Ibn(1,2) ibc(1)
2 4 2	
4 5 3	
5 1 4	Ibn(4,1) Ibn(4,2) ibc(4)

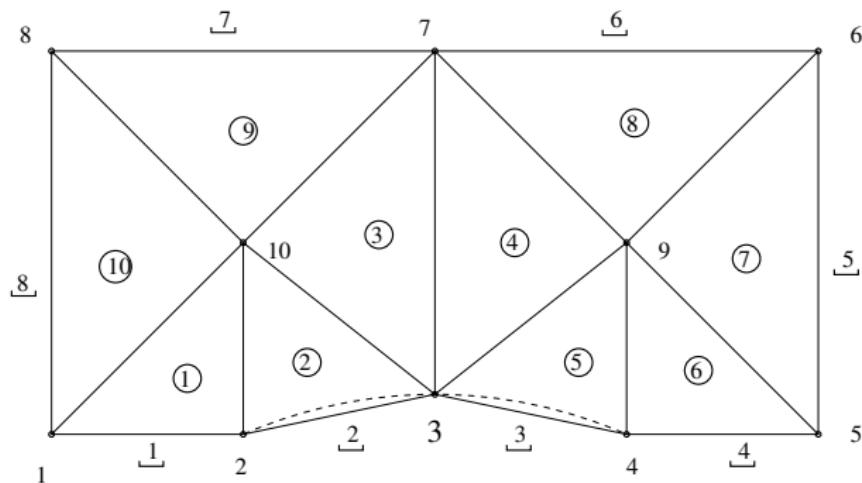
- ° node of triangulation
- 1 index of vertex
- ② index of triangle
- 3 index of boundary segm



Files triang and triangx: GAMM channel

```
10 10 8 4  
0. 0. 0. 0  
-1.0 0.0  
-0.5 0.0  
0.0 0.1  
0.5 0.0  
1.0 0.0  
1.0 1.0  
0.0 1.0  
-1.0 1.0  
0.5 0.5  
-0.5 0.5  
1 2 10  
2 3 10  
3 7 10  
3 9 7  
3 4 9  
4 5 9  
5 6 9  
6 7 9  
7 8 10  
8 1 10  
1 2 3  
2 3 3  
3 4 3  
4 5 3  
5 6 2  
6 7 4  
7 8 4  
8 1 1
```

10 10 8 4	°	node of triangulation
0. 0. 0. 0	1	index of vertex
-1.0 0.0	②	index of triangle
-0.5 0.0	3	index of boundary segment
0.0 0.1		
0.5 0.0		
1.0 0.0		
1.0 1.0		
0.0 1.0		
-1.0 1.0		
0.5 0.5		
-0.5 0.5		
1 2 10		
2 3 10		
3 7 10		
3 9 7		
3 4 9		
4 5 9		
5 6 9		
6 7 9		
7 8 10		
8 1 10		
1 2 3		
2 3 3		
3 4 3		
4 5 3		
5 6 2		
6 7 4		
7 8 4		
8 1 1		



Files result and resultsx

ifv = 0 : cell-vertex, piecewise linear

$$\begin{array}{cccc} w(1, 1) & w(1, 2) & \dots & w(1, ndim) \\ w(2, 1) & w(2, 2) & \dots & w(1, ndim) \\ w(3, 1) & w(3, 2) & \dots & w(3, ndim) \\ \vdots & \vdots & & \vdots \\ w(npoin, 1) & w(npoin, 2) & \dots & w(npoin, ndim) \end{array}$$

ifv = 1 : cell-centered, piecewise constant

$$\begin{array}{cccc} w(1, 1) & w(1, 2) & \dots & w(1, ndim) \\ w(2, 1) & w(2, 2) & \dots & w(1, ndim) \\ w(3, 1) & w(3, 2) & \dots & w(3, ndim) \\ \vdots & \vdots & & \vdots \\ w(nelem, 1) & w(nelem, 2) & \dots & w(nelem, ndim) \end{array}$$

Example of performing the code

Unit square

```
cp triang.01 triang
cp profiles.01 profiles
./side
gnuplot> p 'mesh' w l
```

GAMM channel

```
cp triang.gam triang
cp profiles.gam profiles
./side
gnuplot> p 'mesh' w l
```

Main task 3

Solve numerically the following PDE:

$$\begin{aligned}-\Delta u &= 90x_1^8(1 - x_2^{20}) + 380x_2^{18}(1 - x_1^{10}), \quad \text{in } \Omega = (0, 1)^2, \quad (1) \\ u &= u_D \quad \text{on } \partial\Omega,\end{aligned}$$

where u_D is the exact solution given by

$$u(x_1, x_2) = (1 - x_1^{10})(1 - x_2^{20}), \quad (x_1, x_2) \in \Omega.$$

Instructions

- ① Solve problem (1) by a suitable numerical method and by an arbitrary code based on your choice. You can used freely available software or you can write a simple own code.
- ② Carry out several adaptation cycles using ANGENER.
- ③ Use a suitable visualization of the results, namely the adapted grids.



Combination of codes

- FEM code from Tutorials 11 (sparse) `femP1`
- ANGENER

Comments

- the same structure of files `triang`
- (1) is already implemented in `femP1`
- output of `femP1` in the ANGENER's format `results` is already available
- output of ANGENER: `resultsx` can be used for `femP1` (not necessary)
- careful setting of paramet file: `ityp`, `ifv`

Main task 4

Solve numerically the following PDE:

$$\begin{aligned}-\Delta u &= 0 \quad \text{in } \Omega := (-1, 1) \times (-1, 1) \setminus [0, 1], \\ u &= u_D \quad \text{on } \partial\Omega,\end{aligned}\tag{2}$$

where u_D is the exact solution given by

$$u(r, \phi) = r^{2/3} \sin(2\phi/3), \quad (r, \phi) \in \Omega \text{ are polar coordinates.}$$

Instructions

- ① Using ANGENER, generate a sequence of quasi-uniform grids of Ω and set experimental order of convergence (EOC).
- ② Using the code ANGENER in combination with FEM generate a sequence of adaptively refined grids and set EOC.
- ③ Use a suitable visualization of the adapted grids and the corresponding solutions.