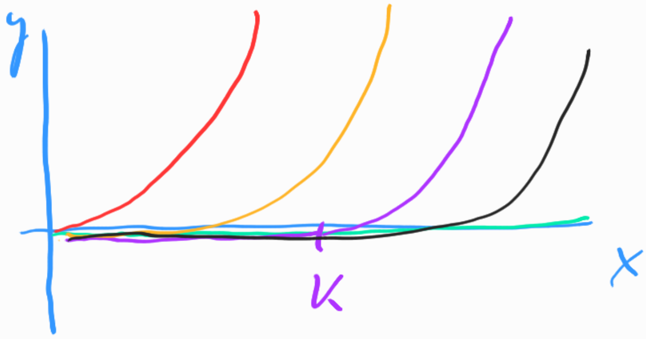
① $\begin{cases} y' = \sqrt{y} \\ y(0) = 0 \end{cases}$ $\quad x \in [0, 5]$




exact: $y = 0$

$y = \frac{1}{4} x^2 \implies \begin{matrix} y' = \frac{x}{2} \\ \sqrt{y} = \frac{x}{2} \end{matrix}$

$y(x) = \begin{cases} 0 & x \le K \\ \frac{(x-K)^2}{4} & x \ge K \end{cases} \quad K \in [0, 5)$

RKF45: $\{y_k\}_{k\ge 0}^{M} \implies$ we obtain only one solution

probably we obtain $y = 0$


$y = \frac{x^2}{4}$

our aim is obtain the non-trivial solution.

One possibility:

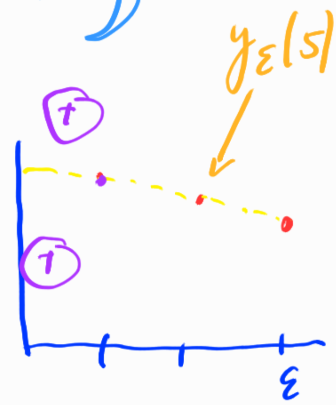
$(*) \begin{cases} y' = \sqrt{y} \\ y(0) = \varepsilon > 0 \end{cases} \implies y_\varepsilon(x) = \left(\frac{x}{2} + \sqrt{\varepsilon}\right)^2$

$y'_\varepsilon(x) = 2\left(\frac{x}{2} + \sqrt{\varepsilon}\right) \cdot \frac{1}{2}$

$\sqrt{y_\varepsilon} = \frac{x}{2} + \sqrt{\varepsilon}$

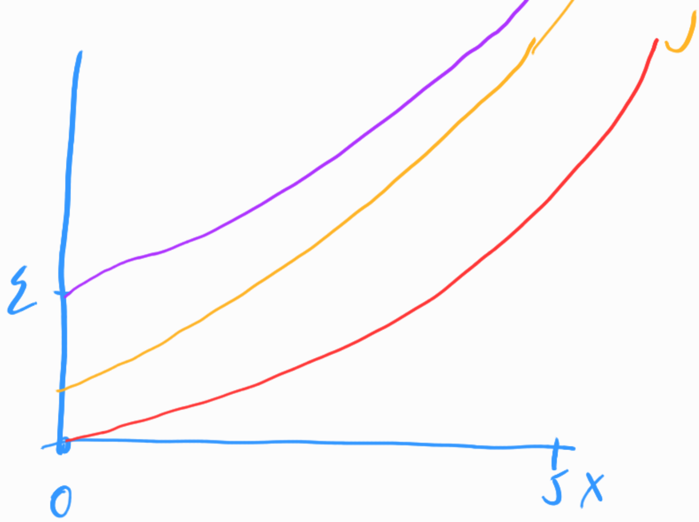
this problem has a unique solution

$\lim_{\varepsilon \to 0^+} y_\varepsilon(x) = y(x) = \frac{x^2}{4}$




Idea of numerical computation:

Solve problem (*) and perform limit $\varepsilon \to 0$




② (P) $\begin{cases} y'' = y \\ y(0) = r \\ y'(0) = s \end{cases}$ $\qquad v_1 = y, \quad v_2 = y' \qquad \begin{cases} v_1' = v_2 \\ v_2' = v_1 \end{cases}$

$y(x) = c_1 e^{x} + c_2 e^{-x}$

$c_1 e^{x} \to \pm\infty$, $\quad c_2 e^{-x} \to 0$

$c_1 = \frac{1}{2}(r+s)$

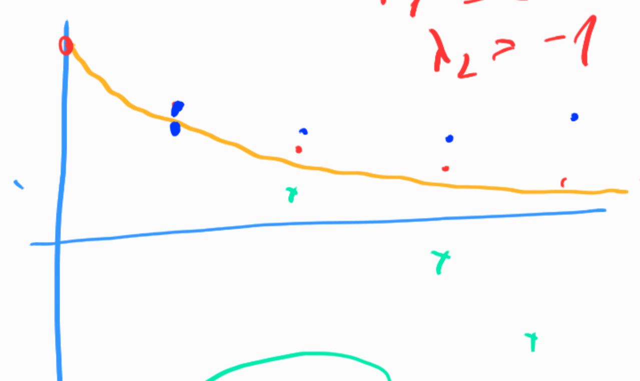
$c_2 = \frac{1}{2}(r-s)$

(P) is unstable

$\lambda_1 = 1$

$\lambda_2 = -1$

- if $c_1 > 0$ $\quad y \to +\infty$
- if $c_1 < 0$ $\quad y \to -\infty$
- if $c_1 = 0$ $\quad y \to 0$



let r and s are given such that $c_1 = 0$

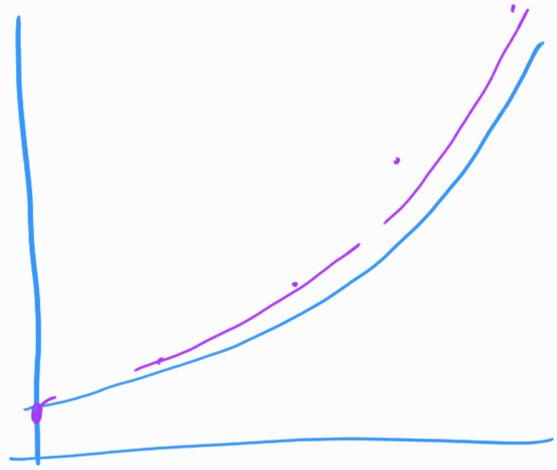
computation is not exact

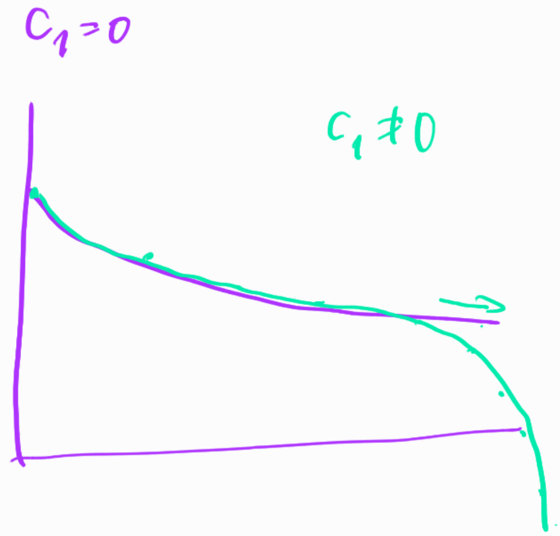
in the second step: $u_1'' = u_1$ $\qquad u_1(x_1) = \dots \neq y(x_1)$

$u_1'(x_1) = \dots \neq y'(x_1)$

solution depends on IC $r, s$ $\Rightarrow c_1, c_2$

$c_1 \neq 0$ solution tends to $\pm\infty$






③ predator-prey problem

fox rabbit

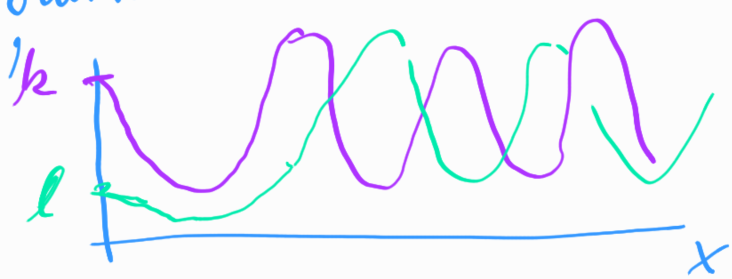
liška $l$ Králík $k$

$$k' = 2k - \alpha k l$$
$$l' = -l + \alpha k l$$

$\alpha > 0$

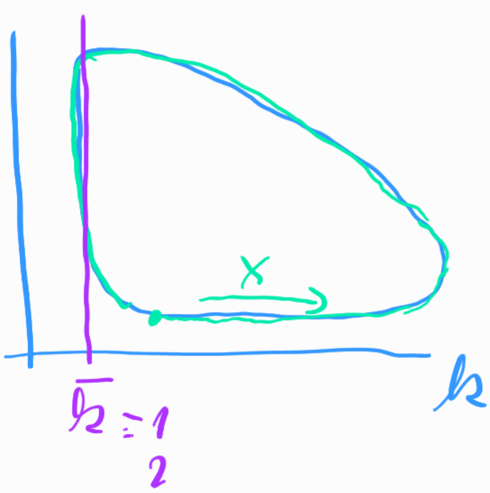
$k(0) = k_0$
$l(0) = l_0$

solution is "periodic"




phase diagram

$l(k)$




When will rabbit die?

$k \leq 1$

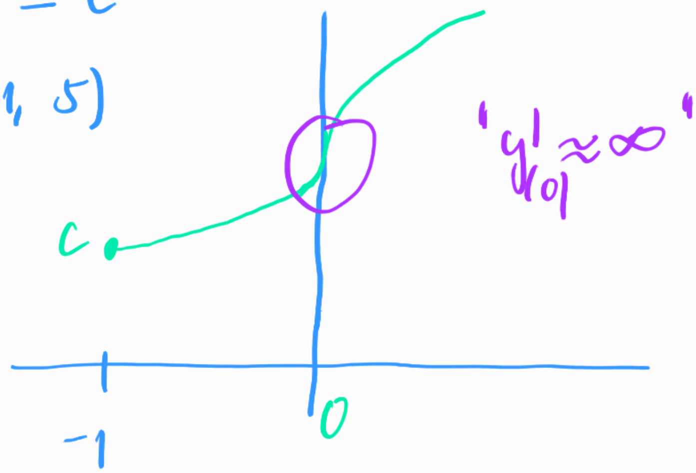
4) $y' = \frac{1}{g(x)}$ $\qquad g(0) = 0 \Rightarrow$ " $\frac{1}{g(x)} = \infty$ "

$y(-1) = c$

$x \in [-1, 5)$

$\int y' = \int \frac{1}{g(x)}$

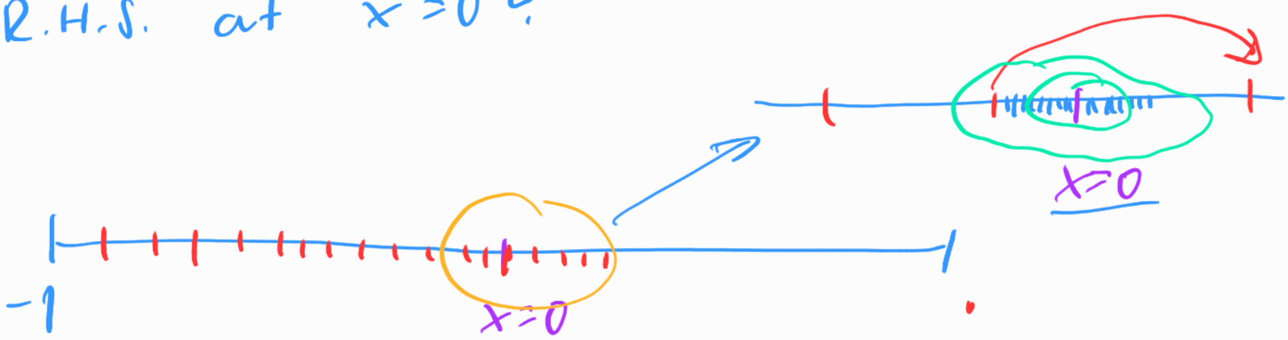
$y(x) - y(-1) =$




Can we expect troubles at $x=0$?

at $x=0$ R.H.S is not defined

Is it really necessary to evaluate R.H.S. at $x=0$?



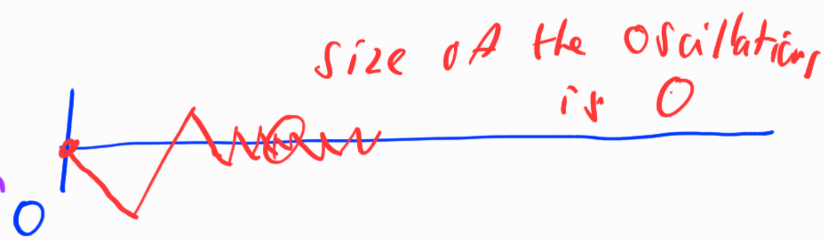

$\Longrightarrow$ it can happends (probably) that we need not to evalute RHS at $x=0$

---

5) $y' = \begin{cases} -1 & \text{if } y \geq 0 \\ 1 & \text{if } y < 0 \end{cases}$

$y(0) = 0$

the problem has no solution




numerical solution? $\{y_k\}$

$y_1' = f(x, y_1)$

$y_2' = 0 \qquad y_2(0) = 0$ } system of 2 equations

$y_1 = y$

$y_2 = 0$

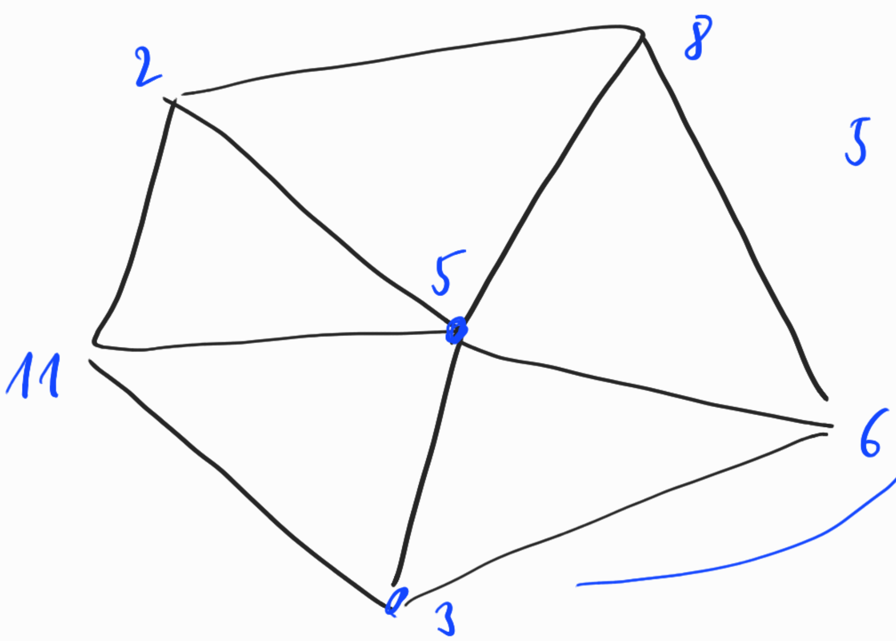
$$\begin{pmatrix} 1 & 0 & 0 & 5 & -3 \\ 7 & 2 & 0 & 0 & 5 \\ -2 & 4 & 0 & 3 & 0 \\ 0 & 0 & 0 & 5 & 6 \\ -1 & 0 & -2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix}$$

npoin $= \cancel{6}\ 5$

nzero $= \underline{14}$

irow(1:7)

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| irow | 1 | 4 | 7 | 10 | 12 | 15 | ~~X~~ | | | | | | | |
| icol | 1 | 4 | 5 | 1 | 2 | 5 | 1 | 2 | 4 | 4 | 5 | 1 | 3 | 5 |
| sparse | 1 | 5 | -3 | 7 | 2 | 5 | -2 | 4 | 3 | 5 | 6 | -1 | -2 | 1 |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |




icol array

5 | 5 | 3 | 6 | 8 | 2 | 11 |

code for sparse matrix operations

UMFPACK