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Canonical affinors on the tangent bundle of a symplectic manifold

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Abstract. All canonical symmetric tensor fields of type $(0, 2)$ on the tangent bundle of a symplectic manifold are classified. As a corollary of this classification and the classification of canonical 2-forms on a symplectic manifold from some our previous paper, all canonical tensor fields of type (1,1) (affinors) on the tangent bundle of a symplectic manifold are described. As a corollary, all canonical 1-forms on the tangent bundle of a symplectic manifold are described. As an application, that symplectic structures do not induce classical linear connections is deduced.

0. Introduction

There are some papers on canonical constructions on symplectic manifolds. For example, in [5], the authors classified all Hamiltonian type natural operators on the cotangent bundle T^*M. In [1], the author described all Hamiltonian type natural operators on a symplectic manifold (M, ω).

Later, in [12], we described all canonical 2-forms $\Lambda(\omega)$ on the tangent bundle TM of a symplectic $2m$-manifold (M, ω). This problem aroused in the context of respective not necessarily regular $\mathcal{M}f_{2m}$-natural operators Λ in the sense of [8]. We proved the following result.

Theorem 1. ([12]) *Let Λ be a not necessarily regular $\mathcal{M}f_{2m}$-natural operator transforming a symplectic structure ω on a $2m$-manifold M into 2-form $\Lambda(\omega)$ on TM. Then there exist real numbers α and β such that*

$$\Lambda(\omega) = \alpha\pi^*\omega + \beta\tilde{\omega}^*\Omega \tag{1}$$

for any symplectic structure ω on M, where $\pi^*\omega$ is the vertical lifting of ω to the tangent bundle TM, $\pi : TM \to M$ is the tangent bundle projection, Ω is the well-known canonical symplectic structure on the cotangent bundle T^*M and $\tilde{\omega} : TM \to T^*M$ is the standard isomorphism induced by ω, $\tilde{\omega}(v) = \omega_x(., v)$, $v \in T_xM$, $x \in M$.

We recall that $\tilde{\omega}^*\Omega$ coincides with the tangent lift $d_T\omega$ by W. Tulczyjew, [20], and (under the isomorphisms $TTM \to T^*TM$ induced by the symplectic form $d_T\omega$) with the tangent lift of the corresponding 2-vector field by J. Grabowski and P. Urbański, [7].

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The present paper deals with canonical constructions on symplectic structures, too. Using Theorem 1, we describe all canonical affinors $\mathbb{A}(\omega): TTM \to TTM$ on the tangent bundle TM of a symplectic $2m$-manifold (M,ω). This problem arises in the concept of respective not necessarily regular $\mathcal{M}f_{2m}$-natural operators in the sense of [8], where $\mathcal{M}f_{2m}$ is the category of $2m$-dimensional manifolds and their local diffeomorphisms. We recall that a not necessarily regular $\mathcal{M}f_{2m}$-natural operator in question is a family of functions

$$\mathbb{A}: SYMP(M) \to AFF(TM)$$

from the set $SYMP(M)$ of all symplectic forms on M into the set $AFF(TM)$ of all affinors on TM for any $2m$-dimensional manifold M satisfying the naturality condition: for any symplectic forms ω_1 on M and ω_2 on N and any local diffeomorphism $\varphi: M \to N$ between $2m$-manifolds if symplectic forms ω_1 and ω_2 are φ-related then affinors $\mathbb{A}(\omega_1)$ and $\mathbb{A}(\omega_2)$ are $T\varphi$-related. The main result of the present note is the following classification theorem.

Theorem 2. *Let $\mathbb{A}$ be a not necessarily regular $\mathcal{M}f_{2m}$-natural operator transforming a symplectic structure ω on a $2m$-manifold M into an affinor $\mathbb{A}(\omega): TTM \to TTM$ on TM. Then there exist uniquely determined real numbers α, β and γ such that*

$$\mathbb{A}(\omega) = \alpha id_{TTM} + \beta J_M + \gamma\tilde{\omega}^*\lambda \otimes L_M \tag{2}$$

for any symplectic structure ω on M, where $id_{TTM}: TTM \to TTM$ is the identity affinor on TM, $J_M: TTM \to TTM$ is the almost tangent structure on TM, $\tilde{\omega}^*\lambda$ is the pull-back of the standard Liouville 1-form λ on the cotangent bundle T^*M by the standard isomorphism $\tilde{\omega}: TM \to T^*M$ induced by ω and L_M is the Liouville vector field on TM.

On the tangent bundle TM of a symplectic $2m$-manifold (M,ω) we have the induced symplectic structure $\tilde{\omega}^*\Omega$ as in Theorem 1. Then affinors on TM can be considered as tensor fields of type $(0,2)$ on TM. Then to prove Theorem 2 it is sufficient to classify all canonical tensor fields $\mathbb{A}(\omega)$ on TM of type $(0,2)$. The skew-symmetric ones form 2-dimensional vector space because of Theorem 1. So, because of the dimension argument to prove Theorem 2 it is sufficient to prove the following theorem.

Theorem 3. *Let $\mathbb{A}$ be a not necessarily regular $\mathcal{M}f_{2m}$-natural operator transforming a symplectic structure ω on a $2m$-manifold M into a symmetric tensor field $\mathbb{A}(\omega)$ of type (0,2) on TM. Then there exists an uniquely determined real number α such that*

$$\mathbb{A}(\omega) = \alpha\tilde{\omega}^*\lambda \odot \tilde{\omega}^*\lambda \tag{3}$$

for any symplectic structure ω on M, where $\tilde{\omega}^*\lambda$ is as in Theorem 2.

The proof of Theorem 3 will occupy Sections 1—11.

Since homotheties are not symplectomorphisms, it is difficult to apply the homogeneous function theorem and the problem of classifications of the operators as in Theorem 3 is more difficult than the one of natural operators defined on all 2-forms, [2], [4], [14], e.t.c.

Using the isomorphism $\tilde{\omega}: TM \to T^*M$ induced by ω we can obtain respective versions of Theorems 1—3 for T^*M instead of TM.

If we have a 1-form $\mathbb{B}(\omega)$ on TM canonically dependent on a symplectic form ω on M, then $\mathbb{B}(\omega) \odot \mathbb{B}(\omega)$ is a symmetric 2-form on TM. By Theorem 3, $\mathbb{B}(\omega) \odot \mathbb{B}(\omega) = \alpha\tilde{\omega}^*\lambda \odot \tilde{\omega}^*\lambda$. Then $\alpha \geqslant 0$ and $\mathbb{B}(\omega) = \underline{+}\alpha^{\frac{1}{2}}\tilde{\omega}^*\lambda$. So, we get

Corollary 1. *Let $\mathbb{B}$ be a not necessarily regular $\mathcal{M}f_{2m}$-natural operator transforming a symplectic structure ω on a $2m$-manifold M into a 1-form $\mathbb{B}(\omega)$ on TM. Then there exists an uniquely determined real number β such that*

$$\mathbb{B}(\omega) = \beta\tilde{\omega}^*\lambda$$

for any symplectic structure ω on M, where $\tilde{\omega}^*\lambda$ is as in Theorem 2.

From the result of M. Paluszny and A. Zajtz [19] it follows that given a symplectic structure (M,ω) there is a torsion free linear ω-connection ∇ on M with $\nabla\omega = 0$. Using Theorem 2 we get that there is no linear connection on M canonically dependent on a symplectic structure (M,ω). So, ∇ is no canonical, in particular, not uniquely determined. In contrast, in the Riemmannian case we have the Levi-Civita connection ∇^g depending canonically on a Riemmannian structure (M,g).

Natural affinors play a very important role in the differential geometry. For example, they can be used to study "generalized" torsions of a connection, see [10], and so they are classified in [3], [6], [9], [11], [13], [15]—[18]. etc. It seems that similar role can play affinors depending canonically on some geometric objects (functions, vector fields, connections, e.t.c.). That is why investigations of natural operators with values in affinors are very useful.

Symplectic structures are involved in the equation of motion. That is why, the results of the paper are interesting with respect to the theoretical mechanics. They are also interesting with respect to the theory of natural operators.

From now on given a map $f : M \to \mathbf{R}$ on a manifold M let $f^{[0]} = f \circ \pi : TM \to \mathbf{R}$ and $f^{[1]} = df : TM \to \mathbf{R}$. The usual coordinates on $\mathbf{R}^{2m}$ are denoted by $x^i, x^{\bar{i}}$, where $i = 1, ..., m$ and $\bar{i} = m + i$. The standard symplectic structure on $\mathbf{R}^{2m}$ is denoted by $\omega_o = \sum_{i=1}^m dx^i \wedge dx^{\bar{i}}$. The system $(x^i)^{[0]}, (x^{\bar{i}})^{[0]}, (x^i)^{[1]}, (x^{\bar{i}})^{[1]}$ for $i = 1, ..., m$ is the induced coordinate system on $T\mathbf{R}^{2m}$.

All manifolds and maps are assumed to be of class $\mathbb{C}^\infty$.

1. A reducibility lemma

Let $\mathbb{A}$ be the not necessarily regular natural operator as in Theorem 3.

Lemma 1. *If $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = 0$, where*

$$T_0\mathbf{R}^m = \{u \in T_0\mathbf{R}^{2m} \mid (x^i)^{[1]}(u) = 0 \text{ for } i = 1, ..., m\} ,$$

then $\Lambda = 0$.

Proof: We have to show that $\Lambda(\omega)(u) = 0$ for any symplectic structure ω on a $2m$-manifold M and any $u \in T_xM$, $x \in M$. By Darboux theorem and the naturality we can assume that $M = \mathbf{R}^{2m}$, $x = 0 \in \mathbf{R}^{2m}$ and $\omega = \omega_o$ is the standard symplectic form. Since $T_0\mathbf{R}^m$ has dense orbit in $T_0\mathbf{R}^{2m}$ with respect to linear ω_o-symplectomorphisms, we can assume that $u \in T_0\mathbf{R}^m$ because of the naturality. □

Because of Lemma 1 $\mathbb{A}$ is uniquely determined by $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$. So, it is sufficient to study $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$.

2. An obvious expression of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$

We can write

$$\begin{aligned} \mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = & \sum_{(k,\lambda)\leqslant(l,\mu)} f_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]})d(x^k)^{[\lambda]} \odot d(x^l)^{[\mu]} + \\ & + \sum_{(k,\lambda)\leqslant(l,\mu)} g_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]})d(x^{\bar{k}})^{[\lambda]} \odot d(x^{\bar{l}})^{[\mu]} + \\ & + \sum_{k,l=1}^{m} \sum_{\lambda,\mu=0,1} h_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]})d(x^k)^{[\lambda]} \odot d(x^{\bar{l}})^{[\mu]} \end{aligned} \tag{4}$$

for some smooth maps $f_{(k,\lambda),(l,\mu)}, g_{(k,\lambda),(l,\mu)}, h_{(k,\lambda),(l,\mu)} : \mathbf{R}^m \to \mathbf{R}$, where the sum $\sum_{(k,\lambda)\leqslant(l,\mu)}$ is over all $(k,\lambda),(l,\mu) \in \{1,...,m\} \times \{0,1\}$ with $(k,\lambda) \leqslant (l,\mu)$ with respect to the lexikographic ordering on $\{1,...,m\} \times \{0,1\}$ (i.e. $k < l$ or ($k = l$ and $\lambda \leqslant \mu$) and $\bar{k} = m + k$, $\bar{l} = m + l$, $\bar{i} = m + i$.

3. Begining of the investigation of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$

Using the invariance of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$ with respect to the ω_o-automorphisms $\overline{x}^i = \tau_i x^i$ and $\overline{x}^{\bar{i}} = \frac{1}{\tau_i}x^{\bar{i}}$ for $i = 1,...,m$ for arbitrary positive real numbers τ_i we get the homogeneity conditions

$$f_{(k,\lambda),(l,\mu)}(\tau_i(x^{\bar{i}})^{[1]}) = \tau_k\tau_l f_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]}) \ ,$$

$$g_{(k,\lambda),(l,\mu)}(\tau_i(x^{\bar{i}})^{[1]}) = \frac{1}{\tau_k}\frac{1}{\tau_l} g_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]})$$

and

$$h_{(k,\lambda),(l,\mu)}(\tau_i(x^{\bar{i}})^{[1]}) = \frac{\tau_k}{\tau_l} h_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]}) \ .$$

Then by the homogeneous function theorem

$$f_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]}) = a_{(k,\lambda),(l,\mu)}(x^{\bar{k}})^{[1]}(x^{\bar{l}})^{[1]} \ ,$$

$$g_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]}) = 0 \ ,$$

$$h_{(k,\lambda),(k,\mu)}((x^{\bar{i}})^{[1]}) = b_{k,\lambda,\mu}$$

and

$$h_{(k,\lambda),(l,\mu)}((x^{\bar{i}})^{[1]}) = 0$$

for $k \neq l$ for some real numbers $a_{(k,\lambda),(l,\mu)}$ and $b_{k,\lambda,\mu}$.

Then we can write

$$\begin{aligned} \mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = & \sum_{(k,\lambda)\leqslant(l,\mu)} a_{(k,\lambda),(l,\mu)}(x^{\bar{k}})^{[1]}(x^{\bar{l}})^{[1]}d(x^k)^{[\lambda]} \odot d(x^l)^{[\mu]} + \\ & + \sum_{k=1}^{m} \sum_{\lambda,\mu=0,1} b_{k,\lambda,\mu} d(x^k)^{[\lambda]} \odot d(x^{\bar{k}})^{[\mu]} \end{aligned} \tag{5}$$

for some real numbers $a_{(k,\lambda),(l,\mu)}$ and $b_{k,\lambda,\mu}$.

4. Some transformation rules

Consider a local $germ_0(\omega_o)$-preserving diffeomorphism

$$\varphi = (x^i - \frac{1}{2}(x^i)^2, \frac{x^{\bar{i}}}{1-x^i}) : U \to \mathbf{R}^{2m} \ ,$$

where U is a neighbourhood of $0 \in \mathbf{R}^{2m}$.

Lemma 2. *For $i = 1, ..., m$ we have*

$$(x^{\bar{i}})^{[1]} \circ T\varphi = (x^{\bar{i}})^{[1]} \tag{6}$$

$$d(x^i)^{[0]} \circ TT\varphi = d(x^i)^{[0]} \ , \quad d(x^{\bar{i}})^{[0]} \circ TT\varphi = d(x^{\bar{i}})^{[0]} \tag{7}$$

$$d(x^i)^{[1]} \circ TT\varphi = d(x^i)^{[1]} \ , \tag{8}$$

$$d(x^{\bar{i}})^{[1]} \circ TT\varphi = d(x^{\bar{i}})^{[1]} + (x^{\bar{i}})^{[1]} d(x^i)^{[0]} \tag{9}$$

on $T_0\mathbf{R}^m$.

Proof. It is clear that

$$(x^i)^{[0]} \circ T\varphi = (x^i)^{[0]} - \frac{1}{2}((x^i)^{[0]})^2 \ ,$$

$$(x^{\bar{i}})^{[0]} \circ T\varphi = \frac{(x^{\bar{i}})^{[0]}}{1-(x^i)^{[0]}} \ ,$$

$$(x^i)^{[1]} \circ T\varphi = (1-(x^i)^{[0]})(x^i)^{[1]} \ ,$$

$$(x^{\bar{i}})^{[1]} \circ T\varphi = \frac{1}{1-(x^i)^{[0]}}(x^{\bar{i}})^{[1]} + \frac{(x^{\bar{i}})^{[0]}}{(1-(x^i)^{[0]})^2}(x^i)^{[1]}$$

over U. Differentiating both sides of these equalities and using the facts that $(x^i)^{[0]} = 0$, $(x^i)^{[1]} = 0$ and $(x^{\bar{i}})^{[0]} = 0$ on $T_0\mathbf{R}^m$ we end the proof. □

5. A conclusion

Using the invariance of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$ with respect to φ from Section 4 and using transformation rules (6)—(9) from (5) we get that $b_{k,\lambda,1} = 0$.

Next using the invariance of $\mathbb{A}(\omega_o)_{|0}$ with respect to the ω_o-automorphisms $\overline{x}^i = x^{\bar{i}}$ and $\overline{x}^{\bar{i}} = -x^i$ from (5) with $(x^{\bar{i}})^{[1]} = 0$ we get that $b_{k,\lambda,\mu} = -b_{k,\mu,\lambda}$.

Consequently, all $b_{k,\lambda,\mu}$ are zero, and we can write

$$\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = \sum_{(k,\lambda)\leqslant(l,\mu)} a_{(k,\lambda),(l,\mu)}(x^{\bar{k}})^{[1]}(x^{\bar{l}})^{[1]} d(x^k)^{[\lambda]} \odot d(x^l)^{[\mu]} \tag{10}$$

for some real numbers $a_{(k,\lambda),(l,\mu)}$.

6. Some transformation rules anew

Consider a local $germ_0(\omega_o)$-preserving diffeomorphism

$$\psi = (\frac{x^i}{1-x^{\bar{i}}}, x^{\bar{i}} - \frac{1}{2}(x^{\bar{i}})^2) : U \to \mathbf{R}^{2m} \ ,$$

where U is a neighbourhood of $0 \in \mathbf{R}^{2m}$.

Lemma 3. *For $i = 1, ..., m$ we have*

$$(x^{\bar{i}})^{[1]} \circ T\varphi = (x^{\bar{i}})^{[1]} \tag{11}$$

$$d(x^i)^{[0]}\circ TT\varphi = d(x^i)^{[0]}\ , \tag{12}$$

$$d(x^i)^{[1]}\circ TT\varphi = d(x^i)^{[1]} + (x^{\bar{i}})^{[1]}d(x^i)^{[0]} \tag{13}$$

on $T_0\mathbf{R}^m$.

Proof: We use the similar arguments as in the proof of Lemma 2. □

7. A conclusion anew

Using the invariance of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$ with respect to ψ from Section 6 and using transformation rules (11)—(13) from (10) we get that $a_{(k,\lambda),(l,\mu)} = 0$ if $\lambda = 1$ or $\mu = 1$.

Consequently, we can write

$$\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = \sum_{k,l=1}^{m} a_{k,l}(x^{\bar{k}})^{[1]}(x^{\bar{l}})^{[1]}d(x^k)^{[0]}\odot d(x^l)^{[0]} \tag{14}$$

for some real numbers $a_{k,l}$.

8. Final expression of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$

Using the invariance of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m}$ with respect to permutations of first m coordinates and respective permutations of the last m coordinates we can write

$$\begin{aligned}\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = 2a \sum_{1\leqslant k<l\leqslant m} &(x^{\bar{k}})^{[1]}(x^{\bar{l}})^{[1]}d(x^k)^{[0]}\odot d(x^l)^{[0]}+\\ &+a\sum_{k=1}^{m}((x^{\bar{k}})^{[1]})^2 d(x^k)^{[0]}\odot d(x^k)^{[0]}+\\ &+b\sum_{k=1}^{m}((x^{\bar{k}})^{[1]})^2 d(x^k)^{[0]}\odot d(x^k)^{[0]}\end{aligned} \tag{15}$$

for some real numbers a and b (with $b = 0$ if $m = 1$).

9. Beginning of the proof of Theorem 3

On replacing $\mathbb{A}(\omega)$ by $\mathbb{A}(\omega) - a\tilde{\omega}^*\lambda\odot\tilde{\omega}^*\lambda$ from (15) we have

$$\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^m} = b\sum_{k=1}^{m}((x^{\bar{k}})^{[1]})^2 d(x^k)^{[0]}\odot d(x^k)^{[0]} \tag{16}$$

for some real number b (with $b = 0$ if $m = 1$). It remains to show that $b = 0$.

10. An expression of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^{2m}}$

Choose real numbers a_i and $c_i \neq 0$ for $i = 1, ..., m$. Let $a = (a_i)$ and $c = (c_i)$ Let $v_{a,c} = \sum_{i=1}^m a_i \frac{\partial}{\partial x^i}_{|0} + \sum_{i=1}^m c_i \frac{\partial}{\partial x^{\bar{i}}}_{|0} \in T_0\mathbf{R}^{2m}$. Because of the naturality of $\mathbb{A}$ with respect to the ω_o-symplectomorphism $\overline{x}^i = x^i - \frac{a_i}{c_i}x^{\bar{i}}$, $\overline{x}^{\bar{i}} = x^{\bar{i}}$ for $i = 1, ..., m$ from (16) at $v_{0,c}$ we obtain

$$\begin{aligned}\mathbb{A}(\omega_o)(v_{a,c}) = b\sum_{k=1}^{m}&(c_k d_{v_{a,c}}(x^k)^{[0]}+\\ -a_k d_{v_{a,c}}(x^{\bar{k}})^{[0]})&\odot(c_k d_{v_{a,c}}(x^k)^{[0]} - a_k d_{v_{a,c}}(x^{\bar{k}})^{[0]})\ .\end{aligned}$$

In other words we can write

$$\mathbb{A}(\omega_o)_{T_0\mathbf{R}^{2m}} = b\sum_{k=1}^{m}((x^{\overline{k}})^{[1]}d(x^k)^{[0]}+$$

$$-(x^k)^{[1]}d(x^{\overline{k}})^{[0]}) \odot ((x^{\overline{k}})^{[1]}d(x^k)^{[0]} - (x^k)^{[1]}d(x^{\overline{k}})^{[0]}) \ . \tag{17}$$

11. End of the proof of Theorem 3

We can assume that $m \geqslant 2$.

We use the invariance of $\mathbb{A}(\omega_o)_{|T_0\mathbf{R}^{2m}}$ with respect to the ω_o-symplectomorphisms $\overline{x}^1 = x^1 + \tau x^{\overline{2}}$, $\overline{x}^{\overline{1}} = x^{\overline{1}}$, $\overline{x}^2 = x^2 + \tau x^{\overline{1}}$, $\overline{x}^{\overline{2}} = x^{\overline{2}}$ and $\overline{x}^i = x^i$ and $\overline{x}^{\overline{i}} = x^{\overline{i}}$ for $i = 3, ..., m$. Then from (17) we obtain the respective equality with both sides being the polynomials in τ. Considering the coefficients on τ^2 we obtain that $b = 0$.

The proof of Theorem 3 is complete. □

12. Non-existence of connections

A generalized connection on a tangent bundle TM is an affinor $\Gamma : TTM \to TTM$ on TM being the projection onto the vertical subbundle VTM. It is a linear connection on M if it is invariant with respect to the fiber homotheties tid_{TM} for $t \neq 0$, [8].

We have the following interesting corollary of Theorem 3.

Corollary 2. *There is no generalized connection Γ^ω on TM canonically dependent on a symplectic structure (M, ω). In particular, there is no linear connection ∇^ω on M canonically dependent on a symplectic structure (M, ω).*

Proof: Suppose that Γ^ω exists. Because of Theorem 3 we can write

$$\Gamma^\omega = \alpha id_{TTM} + \beta J_M + \gamma\tilde{\omega}^*\lambda \otimes L_M \ . \tag{18}$$

Since Γ^ω is a projection onto VTM, $\alpha = 0$. But the rest fragment of (18) is nilpotent. Contradiction. □

References

[1] Debecki, J., *Invariant vector fields of Hamiltonians,* Archivum Math. (Brno), 34(2) (1988), 295-300.

[2] Doupovec, M., *Natural liftings of* $(0, 2)$*-tensor fields to the tangent bundle*, Arch. Math. (Brno) 30(1994),215-225.

[3] Doupovec M., Kolář I., *Natural affinors on time-dependent Weil bundles,* Arch. Math. Brno 27(1991), 205-209.

[4] Doupovec, M., Kurek, J., *Liftings of covariant* $(0, 2)$*-tensor fields to the bundle of* k*-dimensional* 1*-velocities,* Suppl. Rend. Circolo Mat. Palermo, Serie II 43(1996), 111-121.

[5] Doupovec, M., Kurek, J., *Natural operations of Hamiltonian type on the cotangent bundle,* Suppl. Rend. Circolo Mat. Palermo, Serie II 46(1997), 81-86.

[6] Gancarzewicz J., Kolář I., *Natural affinors on the extended r-th order tangent bundles,* Suppl. Rendiconti Circolo Mat. Palermo, 30 (1993), 95-100.

[7] Grabowski, J., Urbański, P., *Tangent lifts of Poisson and related structures*, J. Phys. A 28(1995), 6743-6777.

[8] Kolář I., Michor P. W., Slovák J., *Natural operations in differential geometry,* Springer-Verlag, Berlin 1993.

[9] Kolář I., Mikulski W.M., *Contact elements on fibered manifolds*, Czech. Math. J. 53(128)(2003), 1017-1030.

[10] Kolář I., Modugno M., *Torsions of connections on some natural bundles,* Diff. Geom. and Appl. 2(1992), 1-16.

[11] Kurek J., *Natural affinors on higher order cotangent bundles,* Arch. Math. Brno (28)(1992), 175-180.

[12] Kurek J., Mikulski W.M., *Symplectic structures on the tangent bundles of symplectic and cosymplectic manifolds,* Ann. Polon. Math. 82(3)(2003), 273-285.

[13] Kureš M., Mikulski W.M., *Natural operations lifting vector fields to bundles of Weil contact elements,* Czech. Math. J., to appear.

[14] Mikulski, W.M., *The geometrical constructions liftings tensor fields of type* $(0, 2)$ *on manifolds to the bundles of A-velocities,* Nagoya Math. J. 140(1995), 117-137.

[15] Mikulski W.M., *Natural affinors on r-jet prolongation of the tangent bundles,* Arch. Math. Brno 34(2)(1998), 321-328.

[16] Mikulski W.M., *Natural affinors on* $(J^rT^*)^*$, Archivum Math. Brno 36(2000), 261-267.

[17] Mikulski W.M., *The natural affinors on generalized higher order tangent bundles*, Rend. Mat. Appl. Roma VII 21(2001), 339-349.

[18] Mikulski W.M., *Natural affinors on* $(J^{r,s,q}(., \mathbf{R}^{1,1})_0)^*$, Comment. Math. Univ. Carolinae 42,4(2001), 655-663.

[19] Paluszny M., Zajtz A., *Foundation of the Geometry of natural bundles,* Lecture Notes, Univ. Caracas, 1984.

[20] Tulczyjew, W.M., *Hamiltonian systems, Lagrangian systems, and the Legendre transformation*, Symp. Math. 14 (1974), 101-114.

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