VÝSLEDKY

I. TAYLORŮV POLYNOM

2. a) $x+\frac{1}{3}x^3$, b) $1-\frac{1}{2}x^2+\frac{5}{24}x^4$, c) $x-\frac{1}{3}x^3+\frac{1}{10}x^5$, d) $\frac{x^2}{2}$, e) $1+2x+2x^2-2x^4$
f) $1+2x+x^2-\frac{2}{3}x^3-\frac{5}{6}x^4-\frac{1}{15}x^5$, g) $1-\frac{1}{2}x+\frac{1}{12}x^2-\frac{1}{720}x^4$, h) $-\frac{x^2}{2}-\frac{1}{12}x^4-\frac{1}{45}x^6$
3. a) $\frac{1}{3840}$, b) 2^{-4}
4. 2,236076354981
5. a) $-\frac{1}{12}$, b) $\frac{1}{3}$, c) $\frac{1}{3}$, d) -$\frac{1}{4}$, e) $\log^2 a$, f) 0, g) $\frac{1}{2}$, h) $\frac{1}{3}$, i) $\frac{1}{6}$, j) $\frac{1}{2}$
6. a) AK (absolutně konverguje), b) D (diverguje), c) D, d) AK, e) D, f) AK, g) AK
7. $a=\frac{4}{3}$, $b=-\frac{1}{3}$

II. MOCNINNÉ ŘADY

1. a) R=1, AK pro $|x|<R$, jinak D b) R=$\frac{1}{e}$, AK pro $|x|<R$, jinak D c) R=$\frac{1}{e}$, AK pro $|x|<R$, K pro $x\in[-R,R)$, jinak D d) R=$\frac{1}{3}$, AK pro $x\in(-\frac{4}{3},-\frac{2}{3})$, K pro $x\in[-\frac{4}{3},-\frac{2}{3})$, jinak D e) R=1, AK pro $|x|<R$ a pro $|x|\le R$ pokud $a>1$, jinak D f) R=∞, AK pro $x\in\mathbb{R}$ g) R=$\frac{1}{4}$, AK pro $|x|<R$, jinak D h) R=$\max\{a,b\}$, AK pro $|x|<R$, jinak D i) R=$\min\{\frac{1}{a},\frac{1}{b}\}$. Pokud $b>a$ pak AK pro $|x|\le R$ a jinak D. Pokud $a\ge b$ pak AK pro $|x|<R$, K pro $x\in[-R,R)$ a jinak D. j) R=4, AK pro $|x|<R$, jinak D

III. HLEDÁNÍ PRIMITIVNÍ FUNKCE - ÚVOD

Výsledky jsou uvedeny vždy "až na konstantu":
1. a) $\frac{x^{10}}{10}+\log|x|-5e^x-\frac{1}{2x^2}-\sin x$ na $(-\infty,0)$ a $(0,\infty)$ b) $\frac{2}{3}e^{3x}+\frac{5(5-x)^{\frac{6}{5}}}{6},\ x\in\mathbb{R}$
c) $-\frac{1}{x}-\frac{3}{2x^2}-\frac{2}{x^3}$ na $(-\infty,0)$ a $(0,\infty)$

2. a) $-\log|\cos x|$ na každém z intervalů $(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi),k\in\mathbb{Z}$
b) $\log|\sin x|$ na každém z intervalů $(k\pi,(k+1)\pi),k\in\mathbb{Z}$
c) $\frac{1}{3}\operatorname{tg}x^3$ na každém z intervalů $(-\sqrt[3]{\frac{\pi}{2}+k\pi},\sqrt[3]{\frac{\pi}{2}+k\pi}),k\in\mathbb{Z}$ d) $\frac{1}{2}\operatorname{arctg}x^2,\ x\in\mathbb{R}$
e) $\log|\log x|$ na $(0,1)$ a $(1,\infty)$ f) $\sqrt{x^2+5},\ x\in\mathbb{R}$ g) $\log|\log(\log x)|$ na $(1,e)$ a (e,∞)

3. a) $\frac{4(x^2+7)}{7\sqrt[4]{x}},\ D_f=(0,\infty)$ b) $2x-\frac{12}{5}\sqrt[6]{72x^5}+\frac{3}{2}\sqrt[3]{9x^2},\ D_f=\mathbb{R}\setminus\{0\}$ c) $\frac{4^x}{\log 4}+2\frac{6^x}{\log 6}+\frac{9^x}{\log 9},\ D_f=\mathbb{R}$
d) $x-\operatorname{arctg}x,\ D_f=\mathbb{R}$ e) $-\frac{2}{5}\sqrt{2-5x},\ D_f=(-\infty,\frac{2}{5})$ f) $\frac{1}{4}\operatorname{arctg}(x^4),\ D_f=\mathbb{R}$
g) $\cos(\frac{1}{x}),\ D_f=\mathbb{R}\setminus\{0\}$ h) $\frac{x}{2}+\frac{\sin(2x)}{4},\ D_f=\mathbb{R}$ i) $\operatorname{arctg}(x^2+1),\ D_f=\mathbb{R}$ j) $\frac{1}{2}\operatorname{arctg}(\sin^2 x),\ D_f=\mathbb{R}$

4. a) $-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x,\ x\in\mathbb{R}$ b) $\frac{1}{2}(e^x\sin x+e^x\cos x),\ x\in\mathbb{R}$ c) $x\log x-x,\ x\in(0,\infty)$
d) $I_n:=\int x^ne^x\,\mathrm{d}x=x^ne^x-nI_{n-1};\ I_1:=xe^x-e^x,\ x\in\mathbb{R}$
e) $\frac{1}{4}(2x^2\log x-x^2),\ x\in(0,\infty)$ f) $\frac{1}{2}e^x(x\sin x+x\cos x-\sin x),\ x\in\mathbb{R}$

5. a) $\frac{2}{\sqrt{\cos x}},\ D_f=\bigcup_{k\in\mathbb{Z}}(-\frac{\pi}{2}+2k\pi,\frac{\pi}{2}+2k\pi)$ b) $\frac{1}{8}\sqrt[3]{8x^3+27},\ D_f=\mathbb{R}\setminus\{-\frac{3}{2}\}$ c) $\frac{1}{2}\operatorname{arctg}^2x,\ D_f=\mathbb{R}$
d) $-\frac{1}{2\log\frac{2}{3}}\log|1-(\frac{2}{3})^{2x}|,\ D_f=\mathbb{R}\setminus\{0\}$ e) $x\operatorname{arctg}x-\frac{1}{2}\log(1+x^2),\ D_f=\mathbb{R}$
f) $-\frac{2x^2-1}{4}\cos(2x)+\frac{x}{2}\sin(2x),\ D_f=\mathbb{R}$ g) $\frac{2}{3}x^{3/2}\left(\log^2x-\frac{4}{3}\log x+\frac{8}{9}\right),\ D_f=(0,\infty)$
h) $-\frac{e^{-2x}}{2}(x^2+x+\frac{1}{2}),\ D_f=\mathbb{R}$ i) $-\frac{1}{x}(\log^2x+2\log x+2),\ D_f=(0,\infty)$ j) $\frac{1}{3}(x^3-1)e^{x^3},\ D_f=\mathbb{R}$
k) $2(\sqrt{x}-1)e^{\sqrt{x}},\ D_f=(0,\infty)$ l) $2(6-x)\sqrt{x}\cos\sqrt{x}-6(2-x)\sin\sqrt{x},\ D_f=(0,\infty)$

6. a) $2\arcsin\frac{x}{2}+\sin(2\arcsin\frac{x}{2})=2\arcsin\frac{x}{2}+\frac{x}{2}\sqrt{4-x^2},\ D_f=(-2,2)$
b) $\operatorname{tg}(\arcsin x)=\frac{x}{\sqrt{1-x^2}},\ D_f=(-1,1)$ c) $\frac{1}{a^2}\sin(\operatorname{arctg}\frac{x}{a})=\frac{x}{a^2\sqrt{a^2+x^2}},\ D_f=\mathbb{R}$
d) $a(\arcsin\frac{x}{a}-\cos(\arcsin\frac{x}{a}))=a\arcsin\frac{x}{a}-\sqrt{a^2-x^2},\ x\in(-a,a)$
e) $2a^2(\frac{3\arcsin\sqrt{\frac{x}{2a}}}{2}-\sin(2\arcsin\sqrt{\frac{x}{2a}})+\frac{1}{8}\sin(4\arcsin\sqrt{\frac{x}{2a}})),\ D_f=(0,2a)$

7. a) $\frac{1}{2}|x|x,\ x\in\mathbb{R}$

b) $F(x)=\begin{cases} \sin x+4k & x\in[-\frac{\pi}{2}+2k\pi,\frac{\pi}{2}+2k\pi], k\in\mathbb{Z} \\ -\sin x+4k+2 & x\in(\frac{\pi}{2}+2k\pi,3\frac{\pi}{2}+2k\pi), k\in\mathbb{Z}\end{cases}$ c) $\frac{1}{4}|x|x^3,\ x\in\mathbb{R}$

d) $F(x)=\begin{cases} -\frac{1}{2}\cos(2x-1) & x\geq\frac{1}{2} \\ \frac{1}{2}\cos(2x-1)-1 & x<\frac{1}{2}\end{cases}$

e) $F(x)=(-1)^k(-\cos x+\sin x)+k2\sqrt{2},\ x\in[-\frac{\pi}{4}+k\pi,-\frac{\pi}{4}+(k+1)\pi],\ k\in\mathbb{Z}$

f) $F(x)=\begin{cases} \frac{x^3}{3} & x\in(-\infty,0) \\ \frac{x^2}{2} & x\in[0,1] \\ \frac{x^3}{3}+\frac{1}{6} & x\in(1,\infty)\end{cases}$ g) $F(x)=\begin{cases} e^x-2 & x<0 \\ -e^{-x} & x\geq 0\end{cases}$ h) $F(x)=\begin{cases} -(x^2+x) & x<-\frac{1}{2} \\ x^2+x+\frac{1}{2} & x\geq-\frac{1}{2}\end{cases}$

i) $F(x)=\begin{cases} 2\log(1+x^2)+\pi(x_2-x_1)+4\log\frac{x_1}{x_2} & x\in(-\infty,-x_2] \\ -\pi(x+x_1)-2\log(1+x^2)+4\log(x_1^2+1) & x\in[-x_2,-x_1] \\ 2\log(1+x^2) & x\in[-x_1,x_1] \\ \pi(x-x_1)-2\log(1+x^2)+4\log(x_1^2+1) & x\in[x_1,x_2] \\ 2\log(1+x^2)+\pi(x_2-x_1)+4\log\frac{x_1}{x_2} & x\in[x_2,\infty)\end{cases}$ a $\begin{array}{l} x_1=\frac{4-\sqrt{16-\pi^2}}{\pi} \\ x_2=\frac{4+\sqrt{16-\pi^2}}{\pi}\end{array}$, na $\mathbb{R}$

III. HLEDÁNÍ PRIMITIVNÍ FUNKCE - INTEGRACE RACIONÁLNÍCH FUNKCÍ

Výsledky jsou uvedeny vždy "až na konstantu":

1. a) $\frac{1}{6}\left(\frac{12}{5}\log|x+\frac{3}{2}|+\frac{18}{5}\log|x+\frac{2}{3}|-6\log|x+1|\right),\ D_f=\mathbb{R}\setminus\{-\frac{3}{2},-1,-\frac{2}{3}\}$

b) $5\frac{x^2}{2}-7x+8\log|x+1|+2\frac{1}{x+1},\ D_f=\mathbb{R}\setminus\{-1\}$ c) $x+\log|x-1|-\log|x+1|,\ D_f=\mathbb{R}\setminus\{-1,1\}$

d) $\frac{\sqrt{2}}{8}\log(x^2+\sqrt{2}x+1)+\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x+1)-\frac{\sqrt{2}}{8}\log(x^2-\sqrt{2}x+1)+\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x-1),\ D_f=\mathbb{R}$

e) $\frac{5x+2}{3(x^2+x+1)}+\frac{2}{9}\log|x-1|-\frac{1}{9}\log(x^2+x+1)+\frac{8}{3\sqrt{3}}\operatorname{arctg}(\frac{2x+1}{\sqrt{3}}),\ D_f=\mathbb{R}\setminus\{1\}$

f) $\frac{x^3}{3}+2x^2+10x+20\log|x-1|-15\frac{1}{x-1}-3\frac{1}{(x-1)^2},\ D_f=\mathbb{R}\setminus\{1\}$

2. a) $2\sqrt{x}-2\log(1+\sqrt{x}),\ D_f=(0,\infty)$ [dá se řešit substitucí $t=\sqrt{x}$]

b) $6\sqrt[6]{x+1}-3(\sqrt[6]{x+1})^2-2(\sqrt[6]{x+1})^3+\frac{3}{2}(\sqrt[6]{x+1})^4+\frac{6}{5}(\sqrt[6]{x+1})^5-\frac{6}{7}(\sqrt[6]{x+1})^7+3\log(1+(\sqrt[6]{x+1})^2)-6\operatorname{arctg}(\sqrt[6]{x+1}),\ D_f=(-1,\infty)$ [dá se řešit substitucí $t=\sqrt[6]{x+1}$]

c) $\frac{3}{4}(\sqrt[3]{2+x})^4-\frac{3}{2}(\sqrt[3]{2+x})^2-\frac{3}{4}\log|\sqrt[3]{2+x}-1|+\frac{15}{8}\log((\sqrt[3]{2+x})^2+\sqrt[3]{2+x}+2)-\frac{27}{4\sqrt{7}}\operatorname{arctg}(\frac{2(\sqrt[3]{2+x})+1}{\sqrt{7}}),\ D_f=\mathbb{R}\setminus\{-1\}$ [dá se řešit substitucí $t=\sqrt[3]{2+x}$]

d) $x-\log(1+e^x)+\frac{1}{e^x+1},\ D_f=\mathbb{R}$ e) $-\frac{x}{2}+\frac{1}{3}\log|e^x-1|+\frac{1}{6}\log(e^x+2),\ D_f=\mathbb{R}\setminus\{0\}$

f) $x-3\log(e^{x/6}+1)-3\log(\sqrt{e^{x/3}+1})-3\operatorname{arctg}(e^{x/6}),\ D_f=\mathbb{R}$

g) $\frac{1}{2}\log\left(\frac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}}\right)-\frac{\sqrt{x^2-1}}{x-1+\sqrt{x+1}+2\sqrt{x^2-1}},\ D_f=(1,\infty)$

3. a) $\frac{1}{4}\log\left|\frac{1-\cos x}{1+\cos x}\right|-\frac{1}{2(\cos x+1)},\ D_f=\mathbb{R}\setminus\bigcup_{k\in\mathbb{Z}}\{k\pi,\frac{\pi}{2}+k\pi\}$ [dá se řešit substitucí $t=\cos x$]

b) $\operatorname{tg}x+\log\left|\frac{\operatorname{tg}x}{(\operatorname{tg}x+1)^2}\right|,\ D_f=\mathbb{R}\setminus\bigcup_{k\in\mathbb{Z}}\{k\pi,\frac{\pi}{2}+k\pi,3\frac{\pi}{4}+k\pi\}$ [dá se řešit substitucí $t=\operatorname{tg}x$]

c) $\frac{3}{2}\log(\cos^2 x+1)-\log(\cos^2 x),\ D_f=\mathbb{R}\setminus\bigcup_{k\in\mathbb{Z}}\{\frac{\pi}{2}+k\pi\}$

d) $\frac{1}{3}\operatorname{tg}x+\frac{2}{3\sqrt{3}}\log\left|\frac{\sqrt{3}\operatorname{tg}x-1}{\sqrt{3}\operatorname{tg}x+1}\right|,\ D_f=\mathbb{R}\setminus\bigcup_{k\in\mathbb{Z}}\{\frac{\pi}{2}+k\pi,\frac{\pi}{6}+k\pi,-\frac{\pi}{6}+k\pi\}$

e) $\frac{1}{3}\log(\cos x+2)-\frac{1}{2}\log(\cos x+1)+\frac{1}{6}\log(1-\cos x)=\frac{1}{6}\log\left(\frac{(1-\cos x)(\cos x+2)^2}{(1+\cos x)^3}\right),\ D_f=\mathbb{R}\setminus\bigcup_{k\in\mathbb{Z}}\{k\pi\}$

f) $F(x)=\begin{cases} \operatorname{arctg}(\operatorname{tg}x)-\frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}\operatorname{tg}x)+k\pi(1-1/\sqrt{2}) & x\in(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi),\quad k\in\mathbb{Z} \\ \frac{\pi}{2}-\frac{\pi}{2\sqrt{2}}+k\pi(1-1/\sqrt{2}) & x=\frac{\pi}{2}+k\pi,\quad k\in\mathbb{Z}\end{cases}$

g) $F(x)=\begin{cases} \frac{1}{6}\log\left(\frac{\operatorname{tg}^2x-\operatorname{tg}x+1}{(\operatorname{tg}x+1)^2}\right)+\frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2\operatorname{tg}x-1}{\sqrt{3}}\right) & x\in(-\frac{\pi}{4}+k\pi,\frac{\pi}{2}+k\pi) \\ \frac{\sqrt{3}\pi}{6} & x=\frac{\pi}{2}+k\pi \\ \frac{1}{6}\log\left(\frac{\operatorname{tg}^2x-\operatorname{tg}x+1}{(\operatorname{tg}x+1)^2}\right)+\frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2\operatorname{tg}x-1}{\sqrt{3}}\right)+\frac{\sqrt{3}\pi}{3} & x\in(\frac{\pi}{2}+k\pi,3\frac{\pi}{4}+k\pi)\end{cases}$ pro $k\in\mathbb{Z}$

4. a) $F(x)=\begin{cases} \frac{1}{\sqrt{6}}\operatorname{arctg}\left(\sqrt{\frac{2}{3}}\operatorname{tg}\left(\frac{x}{2}\right)\right)+k\frac{\pi\sqrt{6}}{6} & \text{pro } x\in(-\pi+2k\pi,\pi+2k\pi), k\in\mathbb{Z} \\ \frac{\pi}{2\sqrt{6}}+k\frac{\pi\sqrt{6}}{6} & \text{pro } x=\pi+2k\pi, k\in\mathbb{Z}\end{cases}$

b) $\frac{1}{2}\operatorname{arctg}(\sin^2 x),\ D_f=\mathbb{R}$

c) $F(x) = \begin{cases} \frac{3\sqrt{2}}{8} \operatorname{arctg}\left(\frac{\operatorname{tg} x}{\sqrt{2}}\right) - \frac{\operatorname{tg} x}{4(\operatorname{tg}^2 x + 2)} + k\pi\frac{3\sqrt{2}}{8} & \text{pro } x \in (-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi), k \in \mathbb{Z} \\ \frac{\pi}{2}\frac{3\sqrt{2}}{8} + k\pi\frac{3\sqrt{2}}{8} & \text{pro } x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z} \end{cases}$

d) $F(x) = \begin{cases} \frac{\sqrt{5}}{5} \operatorname{arctg}\left(\frac{3\operatorname{tg}\frac{x}{2} + 1}{\sqrt{5}}\right) + k\pi\frac{\sqrt{5}}{5} & \text{pro } x \in (-\pi + 2k\pi, \pi + 2k\pi), k \in \mathbb{Z} \\ \frac{\pi}{2}\frac{\sqrt{5}}{5} + k\pi\frac{\sqrt{5}}{5} & \text{pro } x = \pi + 2k\pi, k \in \mathbb{Z} \end{cases}$

e) $\frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log(x + \sqrt{x^2+a^2}),\ D_f = \mathbb{R}$

f) $\frac{x}{2}\sqrt{x^2-a^2} - \operatorname{sgn}(x)\frac{a^2}{2}\log(|x| + \sqrt{x^2-a^2}),\ D_f = (-\infty, a] \cup [a, \infty)$

5. a) $\frac{x}{2}\sqrt{x^2-2} + \operatorname{sgn}(x)\log(|x| + \sqrt{x^2-2}),\ D_f = (-\infty, -\sqrt{2}] \cup [\sqrt{2}, \infty)$

b) $\frac{x}{2}\sqrt{x^2+a^2} - \frac{a^2}{2}\log(x + \sqrt{x^2+a^2}),\ D_f = \mathbb{R}$

c) $\frac{2}{x - \sqrt{x^2+2x+2}} - \log(\sqrt{x^2+2x+2} - x - 1),\ D_f = \mathbb{R}$ d) $2\operatorname{sgn}(x-1)\sqrt{\frac{x-2}{x+1}},\ D_f = (-\infty, 1) \cup (2, \infty)$

6. a) $\frac{-2(t^2+3t+1)^2}{(2t+3)^3}$ b) $\frac{-2(t^2-3t+1)^2}{(-2t+3)^3}$ c) $x_1 = \frac{3+\sqrt{5}}{2}$, $x_2 = \frac{3-\sqrt{5}}{2}$. Pak na intervalu $(-\infty, x_2)$ vede na integrál z $\frac{2t^2(x_1-x_2)^2}{(t^2-1)^3}$; na intervalu (x_1, ∞) vede na integrál z $-\frac{2t^2(x_1-x_2)^2}{(t^2-1)^3}$.

7. viz. výsledky ze zkouškových písemek z roku 2005/2006
(pozor, na uvedenych strankach je spatne vysledek prikladu d. Konkretne, pri substituci $t = tg(x/2)$ je ve vysledku spatne znamenko pred logaritmem "$2\log|t-1|$")
`http://www.karlin.mff.cuni.cz/~kalenda/pis-fsv/0506/pismiii.htm`

8. a) $\frac{1}{30}\log(e^x+2) - \frac{1}{2}x + \frac{1}{6}\log|e^x - 1| + \frac{3}{20}\log(e^{2x}+1) - \frac{1}{10}\operatorname{arctg}(e^x),\ D_f = (-3/2, \infty)$

b) $F(x) = \begin{cases} \log\left(\sqrt{\frac{\operatorname{tg}^2\frac{x}{2}+1}{2\operatorname{tg}^2\frac{x}{2} - \operatorname{tg}\frac{x}{2} + 1}}\right) + \operatorname{arctg}(\operatorname{tg}\frac{x}{2}) - \frac{6}{\sqrt{28}}\operatorname{arctg}\left(\frac{8\operatorname{tg}\frac{x}{2} - 2}{\sqrt{28}}\right) - k\pi\left(\frac{6}{\sqrt{28}} - 1\right), & x \in (-\pi + 2k\pi, \pi + 2k\pi), k \in \mathbb{Z} \\ -\frac{1}{2}\log(2) + (-k\pi - \pi/2)\left(\frac{6}{\sqrt{28}} - 1\right), & x = \pi + 2k\pi, k \in \mathbb{Z} \end{cases}$

c) $\frac{1}{2}\log(|2\sqrt{x^2+x} + 2x + 1|) - \frac{1}{2(2\sqrt{x^2+x} + 2x + 1)},\ D_f = (-\infty, -1) \cup (0, \infty)$

d) $\frac{1}{5\sqrt{2}}\operatorname{arctg}(\log(x)/\sqrt{2}) + \frac{9}{10\sqrt{3}}\log\left|\frac{\log(x) - \sqrt{3}}{\log(x) + \sqrt{3}}\right|,\ D_f = (0, \infty) \setminus \{e^{\sqrt{3}}, e^{-\sqrt{3}}\}$

V. NEWTONŮV INTEGRÁL

1. a) 1 b) $\frac{1-\log 2}{2}$ c) 4π d) $2 - \frac{2}{e}$ e) $200\sqrt{2}$ f) $2 - \frac{\pi}{2}$ g) $\frac{a^4}{16}\pi$ h) $\frac{2\pi}{\sqrt{1-\varepsilon^2}}$ i) $\frac{2}{ab}\operatorname{arctg}(\frac{a\operatorname{tg}(1)}{b})$
j) $\frac{\log 3}{2} - \frac{\sqrt{3}}{6}\pi$
k) $2\sqrt{2}\pi$

2. a) $\frac{1}{16}(4 + \sqrt{2}\log(3 + 2\sqrt{2}))$ b) $\frac{\sqrt{3}}{3}\pi$ c) π d) $\frac{5}{2}\sqrt{2}\pi$ e) $\sqrt{2}(1 - 1/3\sqrt{3} - 1/2\log(2\sqrt{3} - 3))$

3. a) $\log 2$ b) $\frac{1}{8}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) + \frac{1}{4}\left(\frac{1}{\sqrt{2}-1} - \frac{1}{(\sqrt{2}+1)^2}\right)$ c) $57(\pi - 2\sqrt{3}\frac{\pi}{3}) + \frac{\sqrt{3}\pi}{9} + \operatorname{arctg}(\operatorname{tg}(18\pi^2)) - \frac{2}{\sqrt{3}}\operatorname{arctg}\left(\frac{2\operatorname{tg}(18\pi^2)+1}{\sqrt{3}}\right)$
d) ∞

4. a) $\frac{1}{3}$ b) $\frac{n-1}{n+1}$ c) 4 d) ∞ e) π

VI. MOCNINNÉ ŘADY - SČÍTÁNÍ ČÍSELNÝCH ŘAD

1. a) $x^5 e^{x^4},\ x \in \mathbb{R}$ b) $\frac{x}{(x-1)^2},\ x \in (-1, 1)$ c) $\log(\frac{1-x}{x}) - \log(1-x) + 1,\ x \in (-1, 1) \setminus \{0\}$, součet je 0 pro $x = 0$ d) $\frac{x(1+x)}{(1-x)^3},\ x \in (-1, 1)$ e) $\frac{x(1-x)}{(1+x)^3},\ x \in (-1, 1)$ f) $\frac{x(3-x)}{(1-x)^3},\ x \in (-1, 1)$ g) $\cosh x,\ x \in \mathbb{R}$ [hint: $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{1+(-1)^n}{n!}x^n$] h) $\frac{1}{2}(\cos x + \cosh x),\ x \in \mathbb{R}$ i) $\frac{x^4(5-3x^2)}{(1-x^2)^3},\ x \in (-1, 1)$
j) $2x\operatorname{arctg} x - \log(1 + x^2),\ x \in (-1, 1)$ k) $\frac{e^{x/3}}{9}(x^2 + 3x - 9) + 1,\ x \in \mathbb{R}$

2. a) $-\log 2$ b) 2 c) $\frac{\pi}{4} - 1$ d) 8 e) $\frac{3}{128}$ f) $2\operatorname{arctg}(1/2) - 1$ g) $\frac{\pi}{4} - \frac{1}{2}$ h) $-2\log(\frac{2}{3}) + \frac{4}{3}$ i) $-\frac{1}{4\sqrt{e}}$

3. a) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!},\ x \in \mathbb{R}$ b) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1},\ x \in (-1, 1)$ c) $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}2^{2n-1}}{(2n)!}x^{2n},\ x \in \mathbb{R}$ d) $\sum_{n=10}^{\infty}x^n$
e) $x + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{n+1}}{n(n+1)}$

VII. KONVERGENCE NEWTONOVA INTEGRÁLU

1. a) konverguje b) diverguje c) konverguje d) konverguje e) diverguje f) konverguje g) konverguje, pokud $p, q < 1$ h) konverguje, pokud $q < 1$ a $p < \frac{1}{2}$ i) konverguje j) konverguje, pokud $p, q > 0$ k) konverguje, pokud $p > -1$

2. a) konverguje, pokud $m < 3$ b) konverguje c) konverguje, pokud $k < -1$ d) konverguje, pokud $\alpha < -1 < \alpha + \beta$ e) konverguje, pokud $\alpha \in (-1, 1)$ f) konverguje, pokud $\alpha + \gamma > -1$ a $\beta - \gamma > -1$ g) konverguje, pokud $p > 1$ nebo $p = 1$ a $q > 1$

4. oba integrály jsou konvergují

5. a) konverguje (neabsolutně) b) konverguje (neabsolutně) c) konverguje, pokud $\alpha \in (1, 3)$ d) neabsolutně konverguje e) konverguje (neabsolutně)

6. a) konverguje (neabsolutně) b) konverguje, pokud $\alpha \in (-2, -1)$ c) konverguje pro $\alpha < 0$ a (neabsolutně) pro $\alpha > 1$

VIII. METRICKÉ PROSTORY

1. a) ano b) ne c) ne

2. a) ano b) ano c) ne d) ne e) ano

3. ano

4. $\frac{1}{4}$

5. a) $\frac{1}{4}$ b) $a = \frac{1}{\sqrt{2}}$

8. a) $\operatorname{Int}(A) = \{(x, y) \in \mathbb{R}^2 : x > 0, y < 0\}$, $\overline{A} = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \leq 0\}$, $H(A) = \{(x, y) \in \mathbb{R}^2 : x = 0, y \leq 0\} \cup \{(x, y) \in \mathbb{R}^2 : x \geq 0, y = 0\}$ b) $\operatorname{Int} B = \emptyset$, $\overline{B} = B$, $H(B) = B$ c) $\operatorname{Int} C = C$, $\overline{C} = \{f \in \mathcal{C}[0, 1] : f(\frac{1}{2}) \in [0, 2]\}$, $H(C) = \{f \in \mathcal{C}[0, 1] : f(\frac{1}{2}) \in \{0, 2\}\}$ d) $\operatorname{Int} D = \emptyset$, $\overline{D} = D$, $H(D) = D$

9. a) platí $\overline{A \cap B} \subset \overline{A} \cap \overline{B}$ ale naopak ne (třeba pro $A = \mathbb{Q}$, $B = \mathbb{R} \setminus \mathbb{Q}$ v $\mathbb{R}$) b) platí $\operatorname{Int}(A \setminus B) \subset \operatorname{Int}(A) \setminus \operatorname{Int}(B)$ ale naopak ne (třeba pro $A = [1, 4]$, $B = [2, 3]$ v $\mathbb{R}$) c) rovnost platí pro A otevřenou

10. všechna tvrzení platí pokud je metrika generovaná normou, jinak ne (jako příklad lze vzít prostor s diskrétní metrikou)

12. pokud není A uzavřená, ekvivalence neplatí (například pro $x = 0$ a $A = (0, 1)$ v $\mathbb{R}$)

IX. FUNKCE VÍCE PROMĚNNÝCH - LIMITY A SPOJITOST

1. a) ano, $f(0, 0) = 2$ b) ano, $f(0, 0) = 0$ c) ano, $f(0, 0) = 0$ d) ne e) ano, $f(x, x) = 3x^2$ f) ano, $f(0, 0) = 0$ g) ne h) ne i) ano, $f(0, 0) = 0$ j) ano, $f(0, 0) = 1$ k) ano, $f(0, 0) = 1$

X. FUNKCE VÍCE PROMĚNNÝCH - DERIVACE

1. a) $\frac{\partial f}{\partial x} = mx^{m-1}y^n$, $\frac{\partial f}{\partial y} = nx^m y^{n-1}$ pro $(x, y) \in \mathbb{R}^2$ b) $\frac{\partial f}{\partial x} = ye^{xy}$, $\frac{\partial f}{\partial y} = xe^{xy}$ pro $(x, y) \in \mathbb{R}^2$ c) $\frac{\partial f}{\partial x} = y+z$, $\frac{\partial f}{\partial y} = x+y$, $\frac{\partial f}{\partial z} = x+y$ pro $(x, y, z) \in \mathbb{R}^3$ d) $\frac{\partial f}{\partial x}(x, y) = |y| \operatorname{sgn} x$ pro $x \neq 0$. $\frac{\partial f}{\partial y}(x, y) = |x| \operatorname{sgn} y$ pro $y \neq 0$. $\frac{\partial f}{\partial x}(0, 0) = \frac{\partial f}{\partial y}(0, 0) = 0$. $\frac{\partial f}{\partial x}(0, y)$ pro $y \neq 0$ a $\frac{\partial f}{\partial y}(x, 0)$ pro $x \neq 0$ neexistují. e) $\frac{\partial f}{\partial x}(x, y) = -\operatorname{sgn}(y+\cos x)\cdot\sin x$, $\frac{\partial f}{\partial y}(x, y) = \operatorname{sgn}(y + \cos x)$, pokud $y \neq -\cos x$. $\frac{\partial f}{\partial y}(x, -\cos x)$ neexistuje pro $x \in \mathbb{R}$. $\frac{\partial f}{\partial x}(k\pi, (-1)^{k+1}) = 0$ pro $k \in \mathbb{Z}$. $\frac{\partial f}{\partial x}(x, -\cos x)$ neexistuje pro $x \neq k\pi$. f) $\frac{\partial f}{\partial x}(x, y) = \cos x \operatorname{sgn}(\sin x - \sin y)$, $\frac{\partial f}{\partial y}(x, y) = -\cos y \operatorname{sgn}(\sin x - \sin y)$, pokud $\sin x \neq \sin y$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = \frac{\partial f}{\partial y}(\frac{\pi}{2} + k\pi, \frac{\pi}{2} + l\pi) = 0$. V ostatních bodech parciální derivace neexistují. g) $\frac{\partial f}{\partial x}(x, y) = -\cos x \operatorname{sgn}(\cos y - \sin x)$, $\frac{\partial f}{\partial y}(x, y) = -\sin y \operatorname{sgn}(\cos y - \sin x)$, pokud $\sin x \neq \cos x$. $\frac{\partial f}{\partial x}(\frac{\pi}{2} + k\pi, (k + 2l)\pi) = \frac{\partial f}{\partial y}(\frac{\pi}{2} + k\pi + 2l\pi, k\pi) = 0$. V ostatních bodech parciální derivace neexistují. h) Pokud $x, y > 0$ nebo $x, y < 0$, pak $\frac{\partial f}{\partial x} = \frac{z}{y} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial y} = -\frac{zx}{y^2} \cdot \left(\frac{x}{y}\right)^{z-1}$; $\frac{\partial f}{\partial z} = \left(\frac{x}{y}\right)^{z} \cdot \log \frac{x}{y}$. i) Pokud $x > 0$ a $z \neq 0$, pak $\frac{\partial f}{\partial x} = \frac{y}{z} \cdot x^{\frac{y}{z}-1}$; $\frac{\partial f}{\partial y} = x^{\frac{y}{z}} \cdot \log x \cdot \frac{1}{z}$; $\frac{\partial f}{\partial z} = -x^{\frac{y}{z}} \cdot \log x \cdot \frac{y}{z^2}$. j) Pokud $x > 0$, pak $\frac{\partial f}{\partial x} = \cos(x^y)\cdot x^{y-1}\cdot y$; $\frac{\partial f}{\partial y} = \cos(x^y)\cdot x^y \cdot \log x$. k) $\frac{\partial f}{\partial x} = e^{-\frac{\pi}{x^2+3xy+3y^2}} \cdot \frac{\pi(2x+3y)}{(x^2+3xy+3y^2)^2}$, $\frac{\partial f}{\partial y} = e^{-\frac{\pi}{x^2+3xy+3y^2}} \cdot \frac{\pi(3x+6y)}{(x^2+3xy+3y^2)^2}$ pro $(x, y) \neq (0, 0)$; v bodě $(0, 0)$ jsou obě parciální derivace nulové. l) Pokud $x > -y^2$, pak $\frac{\partial f}{\partial x} = \frac{1}{2\sqrt{x+y^2}}$;

$\frac{\partial f}{\partial y} = \frac{y}{\sqrt{x+y^2}}$. Jinak parciální derivace nemají smysl. m) Pokud $|x| > |y|$, pak $\frac{\partial f}{\partial x} = \frac{x}{\sqrt{x^2-y^2}}$; $\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{x^2-y^2}}$. V bodech $[x, x]$ a $[x, -x]$ nemá $\frac{\partial f}{\partial y}$ smysl; $\frac{\partial f}{\partial x}$ nemá smysl kromě bodu $[0, 0]$, tam ale neexistuje.

2. a) ne b) ne c) ano d) ne e) ano f) ano g) ano

3. a) $(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2})$ pro $(x, y) \neq (0, 0)$ b) (ye^{xy}, xe^{xy}) pro $(x, y) \in \mathbb{R}^2$ c) $(\frac{1}{y}, -\frac{x}{y^2})$ pro $y \neq 0$

4. a) $1 + mx + ny$ b) $x + y$ **5.**0.345π

6. a) $\bigtriangledown f(x, y) = (-\cos x \cdot \operatorname{sgn}(y - \sin x), \operatorname{sgn}(y - \sin x))$ pro $(x, y) \in \{(x, y) \in \mathbb{R}^2 : y \neq \sin x\}$
b) $\bigtriangledown f(x, y) = (\cos x \cdot \operatorname{sgn}(\sin x - \sin y), -\cos y \cdot \operatorname{sgn}(\sin x - \sin y))$ pro $(x, y) \in \{(x, y) \in \mathbb{R}^2 : \sin y \neq \sin x\}$,
$\bigtriangledown f(x, y) = (0, 0)$ pro $(x, y) \in \{(x, y) \in \mathbb{R}^2 : \sin y = \sin x = \pm 1\}$
c) $\bigtriangledown f(x, y) = \left(\left(\frac{x}{y}\right)^z \cdot \frac{z}{x}, -\left(\frac{x}{y}\right)^z \cdot \frac{z}{y}, \left(\frac{x}{y}\right)^z \cdot \log\left(\frac{x}{y}\right)\right)$, $xy > 0$
d) $\bigtriangledown f(x, y) = \left(x^{\frac{y}{z}} \cdot \frac{y}{zx}, x^{\frac{y}{z}} \frac{\log x}{z}, -x^{\frac{y}{z}} \frac{y \log x}{z^2}\right)$, $x > 0, z \neq 0$
e) $\bigtriangledown f(x, y) = \left(x^{y^z} \cdot \frac{y^z}{x}, x^{y^z} \cdot \frac{zy^z \log x}{y}, x^{y^z} \cdot \log x \cdot y^z \cdot \log y\right)$, $x > 0, y > 0$
f) $\bigtriangledown f(x, y) = \left(e^{-\frac{1}{x^2+xy+y^2}} \cdot \frac{2x+y}{(x^2+xy+y^2)^2}, e^{-\frac{1}{x^2+xy+y^2}} \cdot \frac{2y+x}{(x^2+xy+y^2)^2}\right)$ pro $(x, y) \neq (0, 0)$, $\bigtriangledown f(0, 0) = (0, 0)$

7. a) $\frac{8.9}{3\sqrt[3]{3}} = 2.057$ b) 0.97 c) 1.08 d) $1.06 - \frac{0.07}{12} = 1.0542$ e) $108 \cdot 1.009 = 108.972$