Homework 2 — Multi-step Predictor/Corrector Method

Numerical Solution for ODEs

Due date: December 20, 2019

Support Files

Support files for this homework can be found from the lab computers at:

`V:\Congreve Scott\WS2019_ODE\Homework\2_PredictorCorrector\`

Alternatively, a ZIP file containing these files can be found on:

`http://www.karlin.mff.cuni.cz/~congreve/teaching.php`

Exercises

Exercise 1. Write a MATLAB function with the name `pred_corr` of one of the following *predictor/corrector* methods (see support files for initial template):

Algorithm	*Predictor*	*Corrector*
1. $P(EC)^4$	1-step Adams-Bashfort	3-step Adams-Moulton
2. PEC	2-step Adams-Bashfort	2-step Adams-Moulton
3. $PECE$	3-step Nyström	2-step Milne-Simpson
4. $PECE$	3-step Adams-Bashfort	2-step Adams-Moulton
5. $P(EC)^3E$	1-step Adams-Bashfort	3-step Adams-Moulton
6. $PECE$	2-step Adams-Bashfort	2-step Adams-Moulton
7. $P(EC)^2E$	1-step Adams-Bashfort	3-step Adams-Moulton
8. $P(EC)^2E$	1-step Nyström	2-step Milne-Simpson
9. $PECE$	1-step Adams-Bashfort	2-step Adams-Moulton

Exercise 2. Test your script on the following problems from the support files:

1. The logistic equation $x' = (1 - x)x$ (`logistic.m`) for $t \in [0, 3]$, $x_0 = 2$, $\tau = 0.1$ and plot t versus the solution x:

```
x0=2.0; h=0.1;
figure;
[t,x]=pred_corr('logistic', 0, 3, x0, h);
plot(t,x,'-bx');
```

2. The linear oscillator (`lin_2.m`)

$$x_1' = x_2$$
$$x_2' = -9x_1 + 10\cos(2.5t)$$

for $t \in [0, 10]$, $\boldsymbol{x}_0 = (2, 1)^\top$, $\tau = 0.1$ and plot t versus the solution x_1:

```
figure;
x0 = [2;1]; h = 0.1;
[t,x]=pred_corr('oscil', 0, 10, x0, h);
plot(t,x(:,1),'-bx');
```

3. The satelite problem (`sat_ode.m`) with $\mu = \frac{1}{82.45}$

$$x_1' = x_3$$
$$x_2' = x_4$$
$$x_3' = 2x_4 + x_1 - (1-\mu)\frac{x_1+\mu}{((x_1+\mu)^2 + x_2^2)^{1.5}} - \mu\frac{x_1 - 1 + \mu}{((x_1 - 1 + \mu)^2 + x_2^2)^{1.5}}$$
$$x_4' = -2x_3 + x_2 - (1-\mu)\frac{x_2}{((x_1+\mu)^2 + x_2^2)^{1.5}} - \mu\frac{x_2}{((x_1 - 1 + \mu)^2 + x_2^2)^{1.5}}$$

for $t \in [0, 6.19216933131963970674]$, $\boldsymbol{x}_0 = (1.2, 0, 0, -1.04935750983031990726)^\top$, $\tau = 0.001$ and plot x_1 versus x_2:

```
figure
x0 = [1.2; 0; 0; -1.04935750983031990726]; h = 1e-3;
[t,x] = pred_corr('sat_ode', 0, 6.19216933131963970674, x0, h);
plot(x(:,1), x(:,2));
```

Save each of these plots as a PDF file using `Save > Save As`.

Exercise 3. Apply linear regression to estimate the method order, see `conv_analysis.m` in the support files.

Submission

Email the `pred_corr.m` file from *exercise 1* and PDF files of the plots from *exercise 2* as attachments to

`congreve@karlin.mff.cuni.cz`

including in the title `ODE Homework 2`. Also include in the email the order of the method (*exercise 3*), and which method you have implemented.