Nonlinear Differential Equations

Practical Exercises 1

Due: 28th February 2024

1 Exercises

1. By means of suitable counter-examples show that all the assumptions of the contraction mapping theorem are necessary.

2. For $a > 0$ consider the following gas dynamics model:

$$u(x) = 1 + \frac{1}{\pi} \int_{-1}^{a} \frac{u(y)}{1 + (x - y)^2} \, \mathrm{d}y, \qquad -a \leq x \leq a.$$

Here $u : [-a, a] \to \mathbb{R}$ is an unknown function. Prove that this integral equation has a unique continuous solution u for any $a \in (0, \infty)$. What happens when $a = \infty$?