

## Classical, weak and distributional derivatives

Let us consider the function  $f(x) = \begin{cases} |x|^a \sin\left(\frac{1}{|x|^b}\right), & x \in (-1, 1) \setminus \{0\} \\ 0, & x = 0. \end{cases}$

Find for which values of the parameters  $a, b \in \mathbb{R}$  we have

1.  $f \in L^1((-1, 1))$ ,<sup>1</sup>
2.  $f \in W^{1,1}((-1, 1))$ ,
3.  $f \in W^{1,2}((-1, 1))$ ,
4.  $f$  is differentiable on  $(-1, 1)$  (in the classical sense),
5.  $f \in \mathcal{C}^1((-1, 1))$ .
6. (*Bonus problem, not mandatory*)  $f \in \mathcal{C}^\alpha((-1, 1))$ , where  $\alpha \in (0, 1)$ , resp.  $f \in \mathcal{C}^{0,1}((-1, 1))$ .

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<sup>1</sup>Note that this means that  $f$  has a distributional derivative.