IMPLICITNĚ ZADANÉ FUNKCE

Ukažte, že uvedená rovnice určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}]$ implicitně zadanou funkci[1] φ proměnné x. Spočtěte $\varphi'(\tilde{x})$, $\varphi''(\tilde{x})$.

1. $x^2 + 2xy^2 + y^4 - y^5 = 0, \quad [\tilde{x}, \tilde{y}] = [0, 1]$

2. $e^{xy} + \sin y + y^2 = 1, \quad [\tilde{x}, \tilde{y}] = [2, 0]$

3. $\sin(xy) + \cos(xy) = 1, \quad [\tilde{x}, \tilde{y}] = [\pi, 0]$

4. $2x^4y + x^3 + y^3 + xy = 1, \quad [\tilde{x}, \tilde{y}] = [1, 0]$

5. $\log\left(x^2 + y^2 + \cos(xy)\right) + y = 0, \quad [\tilde{x}, \tilde{y}] = [0, 0]$

6. $\log(x + \operatorname{arctg} y + 1) + xy = 0, \quad [\tilde{x}, \tilde{y}] = [0, 0]$

7. $x^y + y^x = 2y, \quad [\tilde{x}, \tilde{y}] = [1, 1]$

8. $y^3x^2 + y^2x^2 + \sin y = 0, \quad [\tilde{x}, \tilde{y}] = [0, 0]$

9. $e^{\sin x^2} + e^{\sin xy} = 2y + 2, \quad [\tilde{x}, \tilde{y}] = [0, 0]$

10. $\frac{\pi}{2} + \arcsin\left(x + y^2\right) = \arccos\left(y + x^2\right), \quad [\tilde{x}, \tilde{y}] = [0, 0]$

Ukažte, že uvedená rovnice určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{z}]$ implicitně zadanou funkci z proměnných x, y. Spočtěte $\frac{\partial z}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial z}{\partial y}(\tilde{x}, \tilde{y})$.

11. $x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0, \quad [\tilde{x}, \tilde{y}, \tilde{z}] = [1, -2, 1]$

12. $x^2 + y^2 + z^2 - 3xyz = 0, \quad [\tilde{x}, \tilde{y}, \tilde{z}] = [1, 1, 1]$

13. $\frac{x}{z} = \log\frac{z}{y}, \quad [\tilde{x}, \tilde{y}, \tilde{z}] = [0, 1, 1]$

Ukažte, že uvedená soustava rovnic určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}]$ implicitně zadané funkce u, v proměnných x, y. Spočtěte $\frac{\partial u}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial v}{\partial x}(\tilde{x}, \tilde{y})$, $\frac{\partial u}{\partial y}(\tilde{x}, \tilde{y})$, $\frac{\partial v}{\partial y}(\tilde{x}, \tilde{y})$.

14. $xe^{u+v} + 2uv = 1, ye^{u-v} - \frac{u}{1+v} = 2x, \quad [\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [1, 2, 0, 0]$

15. $x = u\cos\frac{v}{u}, y = u\sin\frac{v}{u}, \quad [\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [1, 0, 1, 0]$

16. $x = e^u + u\sin v, y = e^u - u\cos v, \quad [\tilde{x}, \tilde{y}, \tilde{u}, \tilde{v}] = [e + 1, e, 1, \pi/2]$

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[1] tzn. v tomto okolí je splněna rovnice, právě když $y = \varphi(x)$

Ukažte, že uvedená soustava rovnic určuje na nějakém okolí daného bodu $[\tilde{x}, \tilde{y}, \tilde{z}, \tilde{u}, \tilde{v}, \tilde{w}]$ implicitně zadané funkce x, y, z proměnných u, v, w. Spočtěte zadaný gradient.

17. $\begin{cases} u = \operatorname{arctg}(\pi x) + y^2 z, \\ v = e^{-x} + 2\dfrac{y}{z}, \\ w = \cos(2xy) + 2\sqrt{z} \end{cases}$ $\quad [\tilde{u}, \tilde{v}, \tilde{w}] = [4, \frac{3}{2}, 5],$ $\quad [\tilde{x}, \tilde{y}, \tilde{z}] = [0, 1, 4]$ $\quad \nabla y(\tilde{u}, \tilde{v}, \tilde{w}) = \,?$

18. $\begin{cases} u = \sin x + xy + e^z, \\ v = \cos y + xe^{-y}, \\ w = x^2 + 2y - \cos(xz) \end{cases}$ $\quad [\tilde{u}, \tilde{v}, \tilde{w}] = [1 + \sin 1, 2, 0],$ $\quad [\tilde{x}, \tilde{y}, \tilde{z}] = [1, 0, 0]$ $\quad \nabla x(\tilde{u}, \tilde{v}, \tilde{w}) = \,?$

19. $\begin{cases} u = \sin(\pi xy) - \operatorname{arctg}\left(\dfrac{y}{x}\right), \\ v = 3xy^2 + e^{x+y} \end{cases}$ $\quad [\tilde{u}, \tilde{v}] = [\frac{\pi}{4}, -2],$ $\quad [\tilde{x}, \tilde{y}] = [-1, 1]$ $\quad \frac{\partial z}{\partial u}(\tilde{u}, \tilde{v}) = \,?, \text{kde } z(x, y) = x^2 + y^2.$

20. $\begin{cases} u = \operatorname{tg}(xy) + \log\left(x^2 + y^2\right), \\ v = \arcsin(2x) - (3^x)^y \end{cases}$ $\quad [\tilde{u}, \tilde{v}] = [0, -1],$ $\quad [\tilde{x}, \tilde{y}] = [0, 1]$ $\quad \nabla y(\tilde{u}, \tilde{v}) = \,?$

21. $\begin{cases} u = \cos(\pi y) + 2\sqrt{xz}, \\ v = \dfrac{y}{x} - \log z, \\ w = \dfrac{x^2}{2} - \operatorname{arctg}(yz) \end{cases}$ $\quad [\tilde{u}, \tilde{v}, \tilde{w}] = [3, 0, \frac{1}{2}],$ $\quad [\tilde{x}, \tilde{y}, \tilde{z}] = [1, 0, 1]$ $\quad \nabla y(\tilde{u}, \tilde{v}, \tilde{w}) = \,?$

VÝSLEDKY

1. $\varphi'(\tilde{x}) = 2,\ \varphi''(\tilde{x}) = -14$

2. $\varphi'(\tilde{x}) = 0,\ \varphi''(\tilde{x}) = 0$

3. $\varphi'(\tilde{x}) = 0,\ \varphi''(\tilde{x}) = 0$

4. $\varphi'(\tilde{x}) = -1,\ \varphi''(\tilde{x}) = 4$

5. $\varphi'(\tilde{x}) = 0,\ \varphi''(\tilde{x}) = -2$

6. $\varphi'(\tilde{x}) = -1,\ \varphi''(\tilde{x}) = 2$

7. $\varphi'(\tilde{x}) = 1,\ \varphi''(\tilde{x}) = 4$

8. $\varphi'(\tilde{x}) = 0,\ \varphi''(\tilde{x}) = 0$

9. $\varphi'(\tilde{x}) = 0,\ \varphi''(\tilde{x}) = 1$

10. $\varphi'(\tilde{x}) = -1,\ \varphi''(\tilde{x}) = -4$

11. $\frac{\partial z}{\partial x}(\tilde{x},\tilde{y}) = 0,\ \frac{\partial z}{\partial y}(\tilde{x},\tilde{y}) = \frac{7}{5}$

12. $\frac{\partial z}{\partial x}(\tilde{x},\tilde{y}) = -1,\ \frac{\partial z}{\partial y}(\tilde{x},\tilde{y}) = -1$

13. $\frac{\partial z}{\partial x}(\tilde{x},\tilde{y}) = 1,\ \frac{\partial z}{\partial y}(\tilde{x},\tilde{y}) = 1$

14. $\frac{\partial u}{\partial x}(\tilde{x},\tilde{y}) = 0,\ \frac{\partial v}{\partial x}(\tilde{x},\tilde{y}) = -1,$
$\frac{\partial u}{\partial y}(\tilde{x},\tilde{y}) = -1/3,\ \frac{\partial v}{\partial y}(\tilde{x},\tilde{y}) = 1/3$

15. $\frac{\partial u}{\partial x}(\tilde{x},\tilde{y}) = 1,\ \frac{\partial v}{\partial x}(\tilde{x},\tilde{y}) = 0,$
$\frac{\partial u}{\partial y}(\tilde{x},\tilde{y}) = 0,\ \frac{\partial v}{\partial y}(\tilde{x},\tilde{y}) = 1$

16. $\frac{\partial u}{\partial x}(\tilde{x},\tilde{y}) = 1/(1+e),\ \frac{\partial v}{\partial x}(\tilde{x},\tilde{y}) = -e/(1+e),$
$\frac{\partial u}{\partial y}(\tilde{x},\tilde{y}) = 0,\ \frac{\partial v}{\partial y}(\tilde{x},\tilde{y}) = 1$

17. $\nabla y(\tilde{u},\tilde{v},\tilde{w}) = \left(\frac{2}{\pi+16}, \frac{2\pi}{\pi+16}, \frac{\pi-8}{\pi+16}\right),$

18. $\nabla x(\tilde{u},\tilde{v},\tilde{w}) = \left(0, \frac{1}{2}, \frac{1}{4}\right),$

19. $\frac{\partial z}{\partial u}(\tilde{u},\tilde{v}) = -\frac{2}{9-2\pi}$

20. $\nabla y(\tilde{u},\tilde{v}) = \left(\frac{1}{2-\log 3}, \frac{-1}{2(2-\log 3)}\right)$

21. $\nabla x(\tilde{u},\tilde{v},\tilde{w}) = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right),$