Homework 2 – Integer linear programming and VaR

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Computational Aspects of Optimization

Investment problem with VaR

Solve a simple investment problem

$$\min_{x_i} \mathrm{VaR}_\alpha \left(-\sum_{i=1}^{n} x_i R_i \right)$$

$$\text{s.t. } \mathbb{E}\left[\sum_{i=1}^{n} x_i R_i \right] \geq r_0,$$

$$\sum_{i=1}^{n} x_i = 1, \; x_i \geq 0,$$

where we consider n assets with random rate of return R_i. The first constraint ensures minimal expected return r_0, x_i are (nonnegative) portfolio weights which sum to one.

Value at Risk (VaR)

Portfolio optimization problem:

$$\min_{z,x} z$$

$$\begin{aligned} P\left(-\sum_{i=1}^{n} R_i x_i \leq z\right) &\geq \alpha, \\ \sum_{i=1}^{n} \mathbb{E}[R_i] \cdot x_i &\geq r_0, \\ \sum_{i=1}^{n} x_i = 1,\ x_i &\geq 0, \end{aligned}$$

where R_i is random rate of return of $i-$th asset and minimal expected return r_0 is selected in such way that the problem is feasible.

Homework 2

1. Rewrite the VaR minimization problem under a finite discrete distribution as a mixed-integer LP problem (using big-M).
2. Use the same dataset as for the CVaR homework, i.e. at least 6 assets, but the number of scenarios is limited to 50 (if you have free GAMS, otherwise you can use all 100 returns).
3. Consider $\alpha = 0.95$ and run the problem for different 11 values $r_0 \in \{\min_i \overline{R}_i, \ldots, \max_i \overline{R}_i\}$.
4. Plot the optimal values VaR_α against the corresponding values of r_0.