1. Zápočtový test-MA1- (středa 8:10), řešení

1) $\boldsymbol{\lim\limits_{n\to\infty}\left(\frac{(n-1)^{11}-(n+1)^{11}}{44n^2\sqrt{n^{16}+n\ln n-1}}\right)\left(\frac{-\frac{1}{4}+n}{n}\right)^{2n}} =$

$$\lim_{n\to\infty}\left(\frac{(n^{11}-11n^{10}+\cdots+11n-1)-(n^{11}+11n^{10}+\cdots+11n+1)}{44n^2\sqrt{n^{16}(1+\frac{n\ln n}{n^{16}}-\frac{1}{n^{16}})}}\right)\left(\frac{4n-1}{4n}\right)^{2n} =$$

$$\lim_{n\to\infty}\left(\frac{-22n^{10}+An^8+\cdots-2}{44n^{10}\sqrt{1+\frac{\ln n}{n^{15}}-\frac{1}{n^{16}}}}\right)\left(\left(\frac{4n}{4n-1}\right)^{-1}\right)^{2n} = \lim_{n\to\infty}\left(\frac{-22n^{10}(1-\frac{A}{22n^2}+\cdots+\frac{1}{11n^{10}})}{44n^{10}\sqrt{1+\frac{\ln n}{n^{15}}-\frac{1}{n^{16}}}}\right)\lim_{n\to\infty}\left(\left(\frac{4n-1+1}{4n-1}\right)^{2n}\right)^{-1} =$$

$$\frac{-\left(\lim\limits_{n\to\infty}1-\lim\limits_{n\to\infty}\frac{A}{22n^2}+\cdots+\lim\limits_{n\to\infty}\frac{1}{11n^{10}}\right)}{2\sqrt{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{\ln n}{n^{15}}-\lim\limits_{n\to\infty}\frac{1}{n^{16}}}}\lim_{n\to\infty}\left(\left(1+\frac{1}{4n-1}\right)^{4n.\frac{1}{2}}\right)^{-1} = \frac{-(1-0+\cdots+0)}{2\sqrt{1+0-0}}\lim_{n\to\infty}\left(\left(1+\frac{1}{4n-1}\right)^{4n}\right)^{-\frac{1}{2}} =$$

$$\left(\frac{-1}{2}\right)\lim_{n\to\infty}\left(\left(1+\frac{1}{4n-1}\right)^{4n-1+1}\right)^{-\frac{1}{2}} = \frac{-1}{2}\left(\lim_{n\to\infty}\left(1+\frac{1}{4n-1}\right)^{4n-1}\lim_{n\to\infty}\left(1+\frac{1}{4n-1}\right)\right)^{-\frac{1}{2}} =$$

$$\frac{-1}{2}(e.1)^{-\frac{1}{2}} = \boldsymbol{\frac{-1}{2}e^{\frac{-1}{2}}}$$

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2) $\boldsymbol{\lim\limits_{n\to\infty}\frac{n^{100}+8n\ln n+n^2.e^n+n^7}{-n^{\frac{5}{2}}+n\sqrt{n}+3^n}.(n\sqrt{n}-1)} = \lim\limits_{n\to\infty}\frac{n^2.e^n(\frac{n^{100}}{n^2.e^n}+\frac{8n\ln n}{n^2.e^n}+1+\frac{n^7}{n^2.e^n})}{3^n(-\frac{n^{\frac{5}{2}}}{3^n}+\frac{n\sqrt{n}}{3^n}+1)}.(n\sqrt{n}-1) =$

$$\lim_{n\to\infty}\frac{\frac{n^{98}}{e^n}+\frac{8\ln n}{n.e^n}+1+\frac{n^5}{e^n}}{-\frac{n^{\frac{5}{2}}}{3^n}+\frac{n^{\frac{3}{2}}}{3^n}+1}\lim_{n\to\infty}\frac{n^2.e^n}{3^n}(n\sqrt{n}-1) = \left(\frac{\lim\limits_{n\to\infty}\frac{n^{98}}{e^n}+\lim\limits_{n\to\infty}\frac{8\ln n}{n.e^n}+\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{n^5}{e^n}}{-\lim\limits_{n\to\infty}\frac{n^{\frac{5}{2}}}{3^n}+\lim\limits_{n\to\infty}\frac{n^{\frac{3}{2}}}{3^n}+\lim\limits_{n\to\infty}1}\right)\left(\lim_{n\to\infty}\frac{n^2(n\sqrt{n}-1)}{(\frac{3}{e})^n}\right) =$$

$$\left(\frac{0+0+1+0}{-0+0+1}\right)\left(\lim_{n\to\infty}\frac{n^{\frac{7}{2}}-n^2}{(\frac{3}{e})^n}\right) = (1)\left(\lim_{n\to\infty}\frac{n^{\frac{7}{2}}}{\left(\frac{3}{e}\right)^n}-\lim_{n\to\infty}\frac{n^2}{\left(\frac{3}{e}\right)^n}\right) = (1)(0-0) = \mathbf{0}$$

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3) $\boldsymbol{\lim\limits_{n\to\infty}\sqrt[n]{\sqrt{\pi^n+17(3^n)}-\sqrt{3^n+\pi^n}}} = \lim\limits_{n\to\infty}\sqrt[n]{\left(\sqrt{\pi^n+17(3^n)}-\sqrt{3^n+\pi^n}\right).(1)} =$

$$\lim_{n\to\infty}\sqrt[n]{\left(\sqrt{\pi^n+17(3^n)}-\sqrt{3^n+\pi^n}\right).\left(\frac{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}\right)} = \lim_{n\to\infty}\sqrt[n]{\frac{\pi^n+17(3^n)-(3^n+\pi^n)}{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}} =$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{16(3^n)}{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}} = \lim_{n\to\infty}\frac{\sqrt[n]{16(3^n)}}{\sqrt[n]{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}} = \frac{\lim\limits_{n\to\infty}3.\sqrt[n]{16}}{\lim\limits_{n\to\infty}\sqrt[n]{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}}} = \boldsymbol{\frac{3}{\sqrt{\pi}}}$$

$$\sqrt{\pi} = \sqrt[n]{\sqrt{\pi^n}} \le \sqrt[n]{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}} \le \sqrt[n]{\sqrt{\pi^n+\pi^n}+\sqrt{\pi^n+\pi^n}} = \sqrt[n]{2\sqrt{2\pi^n}} = \sqrt[n]{2\sqrt{2}}\sqrt[n]{\sqrt{\pi^n}} = \sqrt[n]{2\sqrt{2}}\sqrt{\pi}$$

$\lim\limits_{n\to\infty}\sqrt{\pi} = \sqrt{\pi}$ $\qquad$ $\lim\limits_{n\to\infty}\sqrt[n]{2\sqrt{2}}\sqrt{\pi} = \sqrt{\pi}\lim\limits_{n\to\infty}\sqrt[n]{2\sqrt{2}} = \sqrt{\pi}$

$$\Rightarrow \lim_{n\to\infty}\sqrt[n]{\sqrt{\pi^n+17(3^n)}+\sqrt{3^n+\pi^n}} = \sqrt{\pi}$$