1. zápočtový test-MA1- (úterý 10:40), řešení

1) $$\lim_{n\to\infty}\left(\frac{(n-1)^{51}-(n+1)^{51}}{102\sqrt{n^{100}-1}}\right)\left(\frac{n-\frac{1}{5}}{n}\right)^{n} = \lim_{n\to\infty}\left(\frac{(n^{51}-51n^{50}+\dots+51n-1)-(n^{51}+51n^{50}+\dots+51n+1)}{102\sqrt{n^{100}(1-\frac{1}{n^{100}})}}\right)\left(\frac{5n-1}{5n}\right)^{n} =$$

$$\lim_{n\to\infty}\left(\frac{-102n^{50}+An^{48}+\dots-2}{102n^{50}\sqrt{1-\frac{1}{n^{100}}}}\right)\left(\left(\frac{5n}{5n-1}\right)^{-1}\right)^{n} =$$

$$\lim_{n\to\infty}\left(\frac{-\cancel{102n^{50}}(1-\frac{A}{102n^{2}}+\dots+\frac{1}{51n^{50}})}{\cancel{102n^{50}}\sqrt{1-\frac{1}{n^{100}}}}\right)\lim_{n\to\infty}\left(\left(\frac{5n-1+1}{5n-1}\right)^{n}\right)^{-1} =$$

$$\frac{-\left(\lim_{n\to\infty}1-\lim_{n\to\infty}\frac{A}{102n^{2}}+\dots+\lim_{n\to\infty}\frac{1}{51n^{50}}\right)}{\sqrt{\lim_{n\to\infty}1-\lim_{n\to\infty}\frac{1}{n^{100}}}}\lim_{n\to\infty}\left(\left(1+\frac{1}{5n-1}\right)^{5n.\frac{1}{5}}\right)^{-1} = \frac{-(1-0+\dots+0)}{\sqrt{1-0}}\lim_{n\to\infty}\left(\left(1+\frac{1}{5n-1}\right)^{5n}\right)^{-\frac{1}{5}} =$$

$$-\lim_{n\to\infty}\left(\left(1+\frac{1}{5n-1}\right)^{5n-1+1}\right)^{-\frac{1}{5}} = -\left(\lim_{n\to\infty}\left(1+\frac{1}{5n-1}\right)^{5n-1}\lim_{n\to\infty}\left(1+\frac{1}{5n-1}\right)\right)^{-\frac{1}{5}} = -(e.1)^{-\frac{1}{5}} =$$

$$\boldsymbol{-e^{\frac{-1}{5}}}$$

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2) $$\lim_{n\to\infty}\left(\sqrt[3]{\frac{1}{n}-8-\frac{2}{n^{3}}}-\sqrt{\frac{1}{n^{3}}+\frac{1}{n!}+1+\frac{1}{n^{n}}}\right)(\sqrt{3n+2}-\sqrt{3n})\sqrt{3n} = \lim_{n\to\infty}\left(\sqrt[3]{\frac{1}{n}-8-\frac{2}{n^{3}}}-\sqrt{\frac{1}{n^{3}}+\frac{1}{n!}+1+\frac{1}{n^{n}}}\right)\lim_{n\to\infty}(\sqrt{3n+2}-\sqrt{3n})\sqrt{3n} =$$

$$\left(\sqrt[3]{\lim_{n\to\infty}\frac{1}{n}-\lim_{n\to\infty}8-\lim_{n\to\infty}\frac{2}{n^{3}}}-\sqrt{\lim_{n\to\infty}\frac{1}{n^{3}}+\lim_{n\to\infty}\frac{1}{n!}+\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{1}{n^{n}}}\right)\lim_{n\to\infty}(\sqrt{3n+2}-\sqrt{3n}).\frac{\sqrt{3n+2}+\sqrt{3n}}{\sqrt{3n+2}+\sqrt{3n}}.\sqrt{3n} = \left(\sqrt[3]{0-8-0}-\sqrt{0+0+1+0}\right)\lim_{n\to\infty}\frac{3n+2-3n}{\sqrt{3n+2}+\sqrt{3n}}.\sqrt{3n} = (-2-1)\lim_{n\to\infty}\frac{2\sqrt{3n}}{\sqrt{3n+2}+\sqrt{3n}} =$$

$$(-2-1)\lim_{n\to\infty}\frac{2\cancel{\sqrt{3n}}}{\cancel{\sqrt{3n}}(\sqrt{1+\frac{2}{3n}}+1)} = -3\frac{\lim_{n\to\infty}2}{\sqrt{\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{2}{3n}}+\lim_{n\to\infty}1} = -3.\frac{2}{\sqrt{1+0}+1} = -3.\frac{2}{2} = \boldsymbol{-3}$$

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3) $$\lim_{n\to\infty}\sqrt[n]{\sqrt{e^{n}+7(2^{n})}-\sqrt{2^{n}+e^{n}}} = \lim_{n\to\infty}\sqrt[n]{\left(\sqrt{e^{n}+7(2^{n})}-\sqrt{2^{n}+e^{n}}\right).(1)} =$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\sqrt{e^{n}+7(2^{n})}-\sqrt{2^{n}+e^{n}}\right).\left(\frac{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}\right)} = \lim_{n\to\infty}\sqrt[n]{\frac{e^{n}+7(2^{n})-(2^{n}+e^{n})}{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}} =$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{6(2^{n})}{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}} = \lim_{n\to\infty}\frac{\sqrt[n]{6(2^{n})}}{\sqrt[n]{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}} = \frac{\lim_{n\to\infty}2.\sqrt[n]{6}}{\lim_{n\to\infty}\sqrt[n]{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}}} = \boldsymbol{\frac{2}{\sqrt{e}}}$$

$$\sqrt{e} = \sqrt[n]{\sqrt{e^{n}}} \le \sqrt[n]{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}} \le \sqrt[n]{\sqrt{e^{n}+e^{n}}+\sqrt{e^{n}+e^{n}}} = \sqrt[n]{2\sqrt{2e^{n}}} = \sqrt[n]{2\sqrt{2}}\sqrt[n]{\sqrt{e^{n}}} = \sqrt[n]{2\sqrt{2}}\sqrt{e}$$

$$\lim_{n\to\infty}\sqrt{e} = \sqrt{e} \qquad\qquad \lim_{n\to\infty}\sqrt[n]{2\sqrt{2}}\sqrt{e} = \sqrt{e}\lim_{n\to\infty}\sqrt[n]{2\sqrt{2}} = \sqrt{e}$$

$$\Rightarrow \lim_{n\to\infty}\sqrt[n]{\sqrt{e^{n}+7(2^{n})}+\sqrt{2^{n}+e^{n}}} = \sqrt{e}$$