1. Zápočtový test-MA1- (úterý 9:00), řešení

1) $$\lim_{n\to\infty}\left(\frac{n-\frac{1}{3}}{n}\right)^{n}\left(\frac{(n-1)^{23}-(n+1)^{23}}{46\sqrt{n^{44}+\ln n-1}}\right)=\lim_{n\to\infty}\left(\frac{3n-1}{3n}\right)^{n}\left(\frac{(n^{23}-23n^{22}+\cdots+23n-1)-(n^{23}+23n^{22}+\cdots+23n+1)}{46\sqrt{n^{44}(1+\frac{\ln n}{n^{44}}-\frac{1}{n^{44}})}}\right)=$$

$$\lim_{n\to\infty}\left((\frac{3n}{3n-1})^{-1}\right)^{n}\left(\frac{-46n^{22}+An^{20}+\cdots-2}{46n^{22}\sqrt{1+\frac{\ln n}{n^{44}}-\frac{1}{n^{44}}}}\right)=\lim_{n\to\infty}\left(\left(\frac{3n-1+1}{3n-1}\right)^{n}\right)^{-1}\lim_{n\to\infty}\left(\frac{-\cancel{46n^{22}}(1-\frac{A}{46n^{2}}+\cdots+\frac{1}{23n^{22}})}{\cancel{46n^{22}}\sqrt{1+\frac{\ln n}{n^{44}}-\frac{1}{n^{44}}}}\right)=$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{3n-1}\right)^{3n.\frac{1}{3}}\right)^{-1}\frac{-\left(\lim\limits_{n\to\infty}1-\lim\limits_{n\to\infty}\frac{A}{46n^{2}}+\cdots+\lim\limits_{n\to\infty}\frac{1}{23n^{22}}\right)}{\sqrt{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{\ln n}{n^{44}}-\lim\limits_{n\to\infty}\frac{1}{n^{44}}}}=\lim_{n\to\infty}\left(\left(1+\frac{1}{3n-1}\right)^{3n}\right)^{-\frac{1}{3}}\frac{-(1-0+\cdots+0)}{\sqrt{1+0-0}}=$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{3n-1}\right)^{3n-1+1}\right)^{-\frac{1}{3}}(-1)=-\left(\lim_{n\to\infty}\left(1+\frac{1}{3n-1}\right)^{3n-1}\lim_{n\to\infty}\left(1+\frac{1}{3n-1}\right)\right)^{-\frac{1}{3}}=$$

$$-(e.1)^{-\frac{1}{3}}=-e^{\frac{-1}{3}}$$

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2) $$\lim_{n\to\infty}\frac{n^{2}+99n\ln n+n.e^{n}+n^{8}}{\sqrt{n}-n^{\frac{3}{2}}+3^{n}}.(\sqrt{n}-1)=\lim_{n\to\infty}\frac{n.e^{n}(\frac{n^{2}}{n.e^{n}}+\frac{99n\ln n}{n.e^{n}}+1+\frac{n^{8}}{n.e^{n}})}{3^{n}(\frac{\sqrt{n}}{3^{n}}-\frac{n^{\frac{3}{2}}}{3^{n}}+1)}.(\sqrt{n}-1)=$$

$$\lim_{n\to\infty}\frac{\frac{n}{e^{n}}+\frac{99\ln n}{e^{n}}+1+\frac{n^{7}}{e^{n}}}{\frac{\sqrt{n}}{3^{n}}-\frac{n^{\frac{3}{2}}}{3^{n}}+1}\lim_{n\to\infty}\frac{n.e^{n}}{3^{n}}(\sqrt{n}-1)=\left(\frac{\lim\limits_{n\to\infty}\frac{n}{e^{n}}+\lim\limits_{n\to\infty}\frac{99\ln n}{e^{n}}+\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{n^{7}}{e^{n}}}{\lim\limits_{n\to\infty}\frac{\sqrt{n}}{3^{n}}-\lim\limits_{n\to\infty}\frac{n^{\frac{3}{2}}}{3^{n}}+\lim\limits_{n\to\infty}1}\right)\left(\lim_{n\to\infty}\frac{n(\sqrt{n}-1)}{(\frac{3}{e})^{n}}\right)=$$

$$\left(\frac{0+0+1+0}{0-0+1}\right)\left(\lim_{n\to\infty}\frac{n^{\frac{3}{2}}-n}{(\frac{3}{e})^{n}}\right)=(1)\left(\lim_{n\to\infty}\frac{n^{\frac{3}{2}}}{\left(\frac{3}{e}\right)^{n}}-\lim_{n\to\infty}\frac{n}{\left(\frac{3}{e}\right)^{n}}\right)=(1)(0-0)=\mathbf{0}$$

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3) $$\lim_{n\to\infty}\sqrt[n]{\sqrt{\pi^{n}+5(2^{n})}-\sqrt{2^{n}+\pi^{n}}}=\lim_{n\to\infty}\sqrt[n]{\left(\sqrt{\pi^{n}+5(2^{n})}-\sqrt{2^{n}+\pi^{n}}\right).(1)}=$$

$$\lim_{n\to\infty}\sqrt[n]{\left(\sqrt{\pi^{n}+5(2^{n})}-\sqrt{2^{n}+\pi^{n}}\right).\left(\frac{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}\right)}=\lim_{n\to\infty}\sqrt[n]{\frac{\pi^{n}+5(2^{n})-(2^{n}+\pi^{n})}{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}}=$$

$$\lim_{n\to\infty}\sqrt[n]{\frac{4(2^{n})}{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}}=\lim_{n\to\infty}\frac{\sqrt[n]{4(2^{n})}}{\sqrt[n]{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}}=\frac{\lim\limits_{n\to\infty}2.\sqrt[n]{4}}{\lim\limits_{n\to\infty}\sqrt[n]{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}}=\frac{\mathbf{2}}{\sqrt{\boldsymbol{\pi}}}$$

$$\sqrt{\pi}=\sqrt[n]{\sqrt{\pi^{n}}}\le\sqrt[n]{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}\le\sqrt[n]{\sqrt{\pi^{n}+\pi^{n}}+\sqrt{\pi^{n}+\pi^{n}}}=\sqrt[n]{2\sqrt{2\pi^{n}}}=\sqrt[n]{2\sqrt{2}}\sqrt[n]{\sqrt{\pi^{n}}}=\sqrt[n]{2\sqrt{2}}\sqrt{\pi}$$

$$\lim_{n\to\infty}\sqrt{\pi}=\sqrt{\pi}\qquad\qquad\lim_{n\to\infty}\sqrt[n]{2\sqrt{2}}\sqrt{\pi}=\sqrt{\pi}\lim_{n\to\infty}\sqrt[n]{2\sqrt{2}}=\sqrt{\pi},$$

$$\Rightarrow\lim_{n\to\infty}\sqrt[n]{\sqrt{\pi^{n}+5(2^{n})}+\sqrt{2^{n}+\pi^{n}}}=\sqrt{\pi}$$