Předmět: NMTM101 Matematická analýza I

Typ výuky: Cvičení

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Cvičení 8

$$\lim_{x\to 0}\frac{(1+x)(1+2x)(1+3x)\dots(1+nx)-1}{x}$$

$$=\lim_{x\to 0}\frac{Ax^n+Bx^{n-1}+\cdots Cx+\cancel{1-1}}{x}=\lim_{x\to 0}\frac{\cancel{x}(Ax^{n-1}+Bx^{n-2}+\cdots+C)}{\cancel{x}}=C=\frac{n(n+1)}{2}$$

$$(1+x)(1+2x)(1+3x)\dots(1+nx)$$

$$x.1.1\dots1=x$$

$$1.2x.1.\dots1=2x$$

$$1.1.3x.\dots1=3x \qquad \Rightarrow\ x+2x+3x+\cdots+nx=(1+2+\cdots+n)x=\frac{n(n+1)}{2}x$$

$$\dots$$

$$1.1.1.\dots nx=nx$$

$$\lim_{x\to1}\frac{x^3-3x+2}{x^4-4x+3} \quad \frac{0}{0}$$

$$\boxed{(x-1)}$$

$$x^3-3x+2=(x-1)(x^2+x-2)$$

$$x^4-4x+3=(x-1)(x^3+x^2+x-3)$$

$$x^3-3x+2 \div (x-1) = x^2+x-2$$

$$\begin{array}{l} x^3-3x+2 \\ -\ x^3-x^2 \\ \hline x^2-3x+2 \\ -\ x^2-x \\ \hline -2x+2 \\ -\ -2x+2 \\ \hline 0 \end{array}$$

$$\lim_{x\to1}\frac{x^3-3x+2}{x^4-4x+3}=\lim_{x\to1}\frac{\cancel{(x-1)}(x^2+x-2)}{\cancel{(x-1)}(x^3+x^2+x-3)} \quad \frac{0}{0}$$

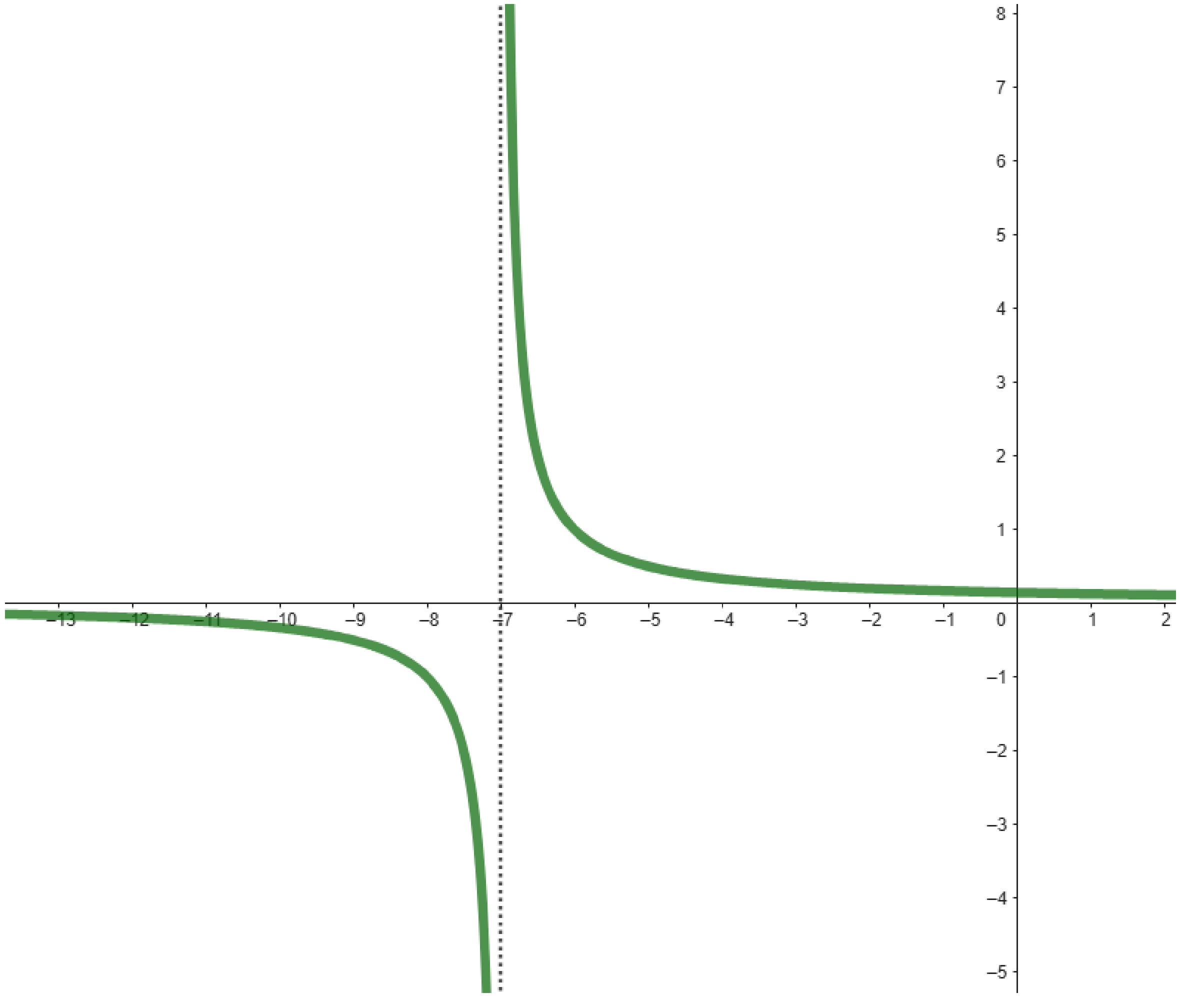
$$=\lim_{x\to1}\frac{\cancel{(x-1)}(x+2)}{\cancel{(x-1)}(x^2+2x+3)}=\frac{3}{6}=\frac{1}{2}$$

$$\lim_{x\to -7} \frac{x+7}{x^2+14x+49} \qquad \frac{0}{0}$$

$$= \lim_{x\to -7} \frac{x+7}{(x+7)^2} = \lim_{x\to -7} \frac{1}{x+7} \qquad \textbf{neexistuje}$$

$$\lim_{x\to -7^-} \frac{1}{x+7} = -\infty$$

$$\lim_{x\to -7^+} \frac{1}{x+7} = +\infty$$



$$\lim_{x \to 1} \frac{x^{100} - 2x + 1}{x^{50} - 2x + 1} \qquad \frac{0}{0} \qquad (x - 1)$$

$$A^n - B^n = (A - B)(A^{n-1}B^0 + A^{n-2}B^1 + \dots + A^1B^{n-2} + A^0B^{n-1})$$

$$x^{100} - 2x + 1 = (x - 1).(x^{99} + x^{98} + x^{97} + \cdots + x - 1)$$

$$x^{50} - 2x + 1 = (x - 1).(x^{49} + x^{48} + x^{47} + \cdots + x - 1)$$

$$= \lim_{x \to 1} \frac{\cancel{(x - 1)}(x^{99} + x^{98} + x^{97} + \cdots + x - 1)}{\cancel{(x - 1)}(x^{49} + x^{48} + x^{47} + \cdots + x - 1)} = \frac{99 - 1}{49 - 1} = \frac{98}{48} = \frac{49}{24}$$

$$(x-1)$$

$$\lim_{x \to 1} \frac{x^n - 1}{x^m - 1} \qquad \frac{0}{0}$$

$$A^n - B^n = (A-B)(A^{n-1}B^0 + A^{n-2}B^1 + \ldots + A^1B^{n-2} + A^0B^{n-1})$$

$$x^n - 1 = (x-1)\left(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + x + 1\right)$$

$$x^m - 1 = (x-1)\left(x^{m-1} + x^{m-2} + x^{m-3} + \cdots + x + 1\right)$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}\left(x^{n-1} + x^{n-2} + x^{n-3} + \cdots + x + 1\right)}{\cancel{(x-1)}\left(x^{m-1} + x^{m-2} + x^{m-3} + \cdots + x + 1\right)} = \frac{n}{m}$$

$$(x-1)$$

$$\lim_{x\to 1}\frac{x+x^2+x^3+\cdots+x^n-n}{x-1} \qquad \frac{0}{0}$$

$$=\lim_{x\to 1}\frac{x+x^2+x^3+\cdots+x^n-1-1-1\ldots-1}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)+(x^2-1)+(x^3-1)\ldots+(x^n-1)}{x-1}$$

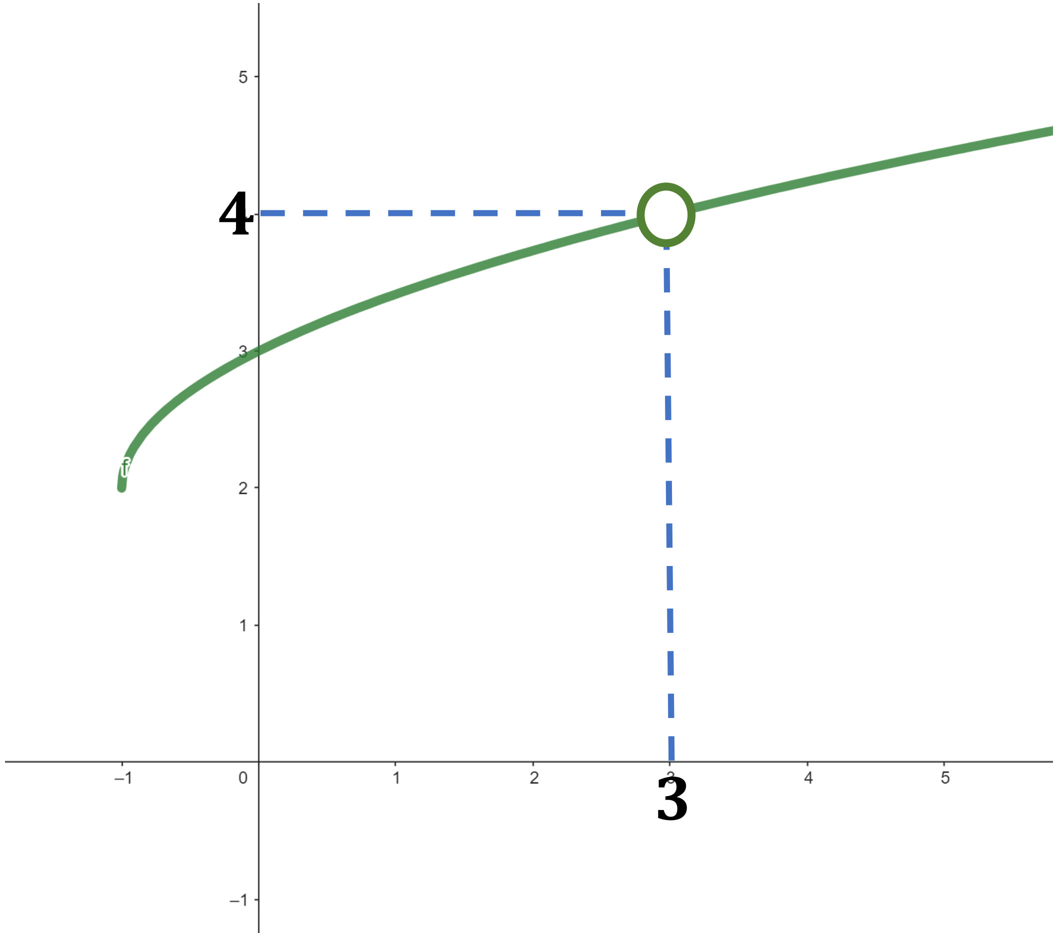
$$=\lim_{x\to 1}\frac{\cancel{(x-1)}+\cancel{(x-1)}(x+1)+\cancel{(x-1)}(x^2+x+1)\ \ldots+\cancel{(x-1)}(x^{n-1}+\cdots+x+1)}{\cancel{x-1}}$$

$$=1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x+1}-2}\qquad \frac{0}{0}$$

$$f(x)=\frac{x-3}{\sqrt{x+1}-2}\qquad g(x)=\sqrt{x+1}+2$$

$$=\lim_{x\to 3}\frac{(x-3)(\sqrt{x+1}+2)}{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}=\lim_{x\to 3}\frac{\cancel{(x-3)}(\sqrt{x+1}+2)}{\cancel{x+1-4}}=4$$



$$\lim_{x\to 16}\frac{\sqrt[4]{x}-2}{\sqrt{x}-4} \qquad \frac{0}{0}$$

$$\boxed{(A-B)\left(A^3+A^2B+AB^2+B^3\right)=A^4-B^4}$$

$$A=\sqrt[4]{x}\,,\ B=2$$

$$=\lim_{x\to 16}\frac{\sqrt[4]{x}-2}{\sqrt{x}-4}.1.1=\lim_{x\to 16}\frac{(\sqrt[4]{x}-2)}{(\sqrt{x}-4)}.\frac{(\sqrt{x}+4)}{(\sqrt{x}+4)}.\frac{(A^3+A^2B+AB^2+B^3)}{(A^3+A^2B+AB^2+B^3)}$$

$$=\lim_{x\to 16}\frac{\left(\left(\sqrt[4]{x}\right)^4-2^4\right)(\sqrt{x}+4)}{(x-16)(A^3+A^2B+AB^2+B^3)}=\lim_{x\to 16}\frac{\cancel{(x-16)}(\sqrt{x}+4)}{\cancel{(x-16)}(A^3+A^2B+AB^2+B^3)}$$

$$=\lim_{x\to 16}\frac{\sqrt{x}+4}{(\sqrt[4]{x})^3+\left(\sqrt[4]{x}\right)^2(2)+(\sqrt[4]{x})(2)^2+(2)^3}=\frac{8}{8+8+8+8}=\frac{1}{4}$$

$$\lim_{x\to 16} \frac{\sqrt[4]{x}-2}{\sqrt{x}-4} \qquad \frac{0}{0}$$

$$= \lim_{x\to 16} \frac{\cancel{(\sqrt[4]{x}-2)}}{\cancel{(\sqrt[4]{x}-2)}(\sqrt[4]{x}+2)} = \frac{1}{2+2} = \frac{1}{4}$$