Předmět: NMTM101 Matematická analýza I

Typ výuky: Cvičení

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Cvičení 1

V oboru reálných čísel řešte následující rovnice a nerovnice.

$3^{2x} - 3^x - 6 = 0$

$3^x = y$

$(3^x)^2 - (3^x) - 6 = 0 \quad \Rightarrow y^2 - y - 6 = 0 \quad \Rightarrow (y-3)(y+2) = 0$

$\Rightarrow y = 3 \quad \vee \quad y = -2$

(nebo)

$\Rightarrow 3^x = 3 \quad \vee \quad 3^x = -2$

$x = 1$ ✓

$3^x = -2$ ✗

$$y_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1 \quad b = -1 \quad c = -6$

$$y_{1,2} = \frac{1 \pm \sqrt{1 + 24}}{2}$$

$y = 3$

$y = -2$

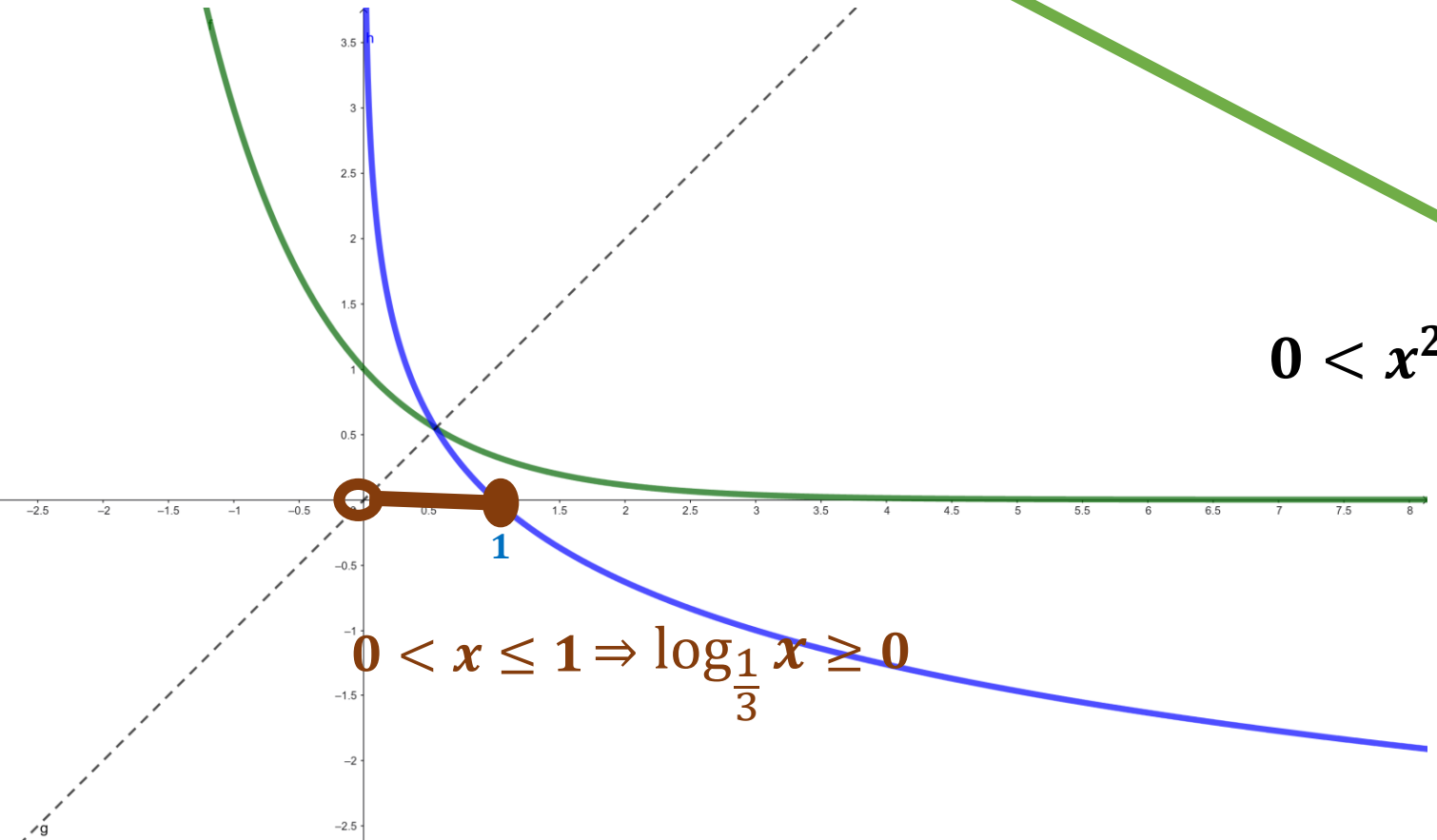
$$\log_{\frac{1}{3}}(x^2-3x+3)\geq 0$$

$$f(x)=\log_a x \qquad 0<a<1 \qquad D_f=(0,+\infty)$$

$$f(x)=\log_{\frac{1}{3}} x \qquad \boxed{a=\frac{1}{3}}$$

$$f=a^x \quad \overset{\text{inverzní}}{\Leftrightarrow} \quad f^{-1}=\log_a x$$

$$f=\left(\frac{1}{3}\right)^x \quad \overset{\text{inverzní}}{\Leftrightarrow} \quad f^{-1}=\log_{\frac{1}{3}} x$$



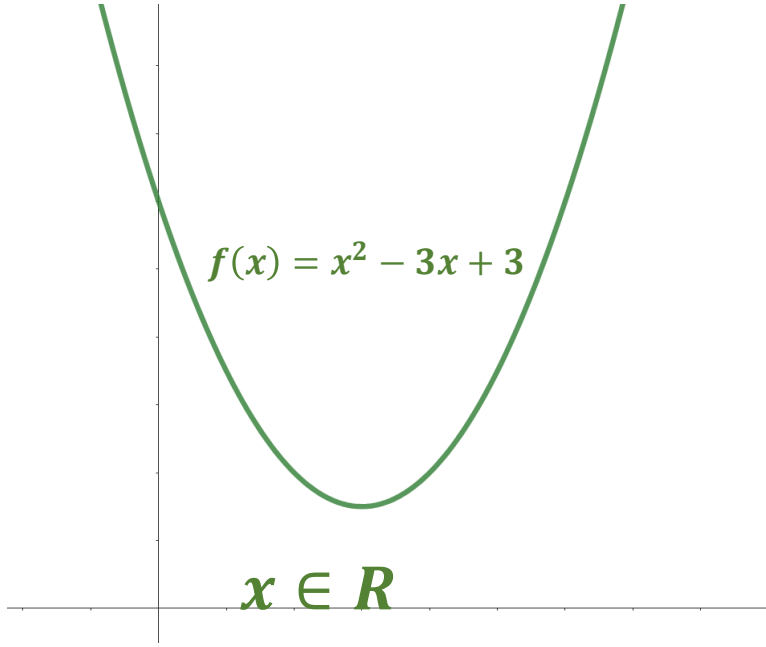
$$0<x\leq 1 \Rightarrow \log_{\frac{1}{3}} x\geq 0$$

$$0<x^2-3x+3\leq 1 \quad \begin{cases} 0<x^2-3x+3 \\ \wedge \\ x^2-3x+3\leq 1 \end{cases}$$

$$\log_{\frac{1}{3}}(x^2 - 3x + 3) \geq 0$$

$0 < x^2 - 3x + 3$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x_{1,2} = \frac{3 \pm \sqrt{9 - 12}}{2}$$

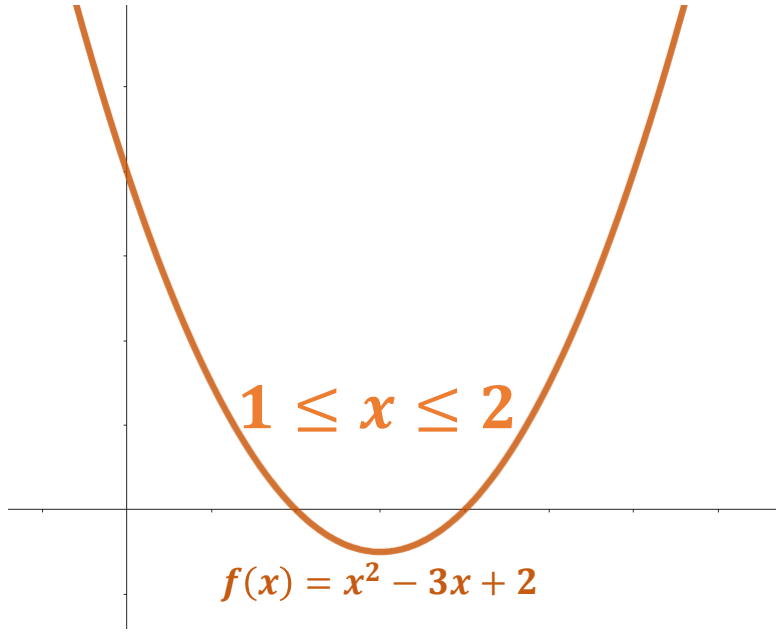


$x^2 - 3x + 3 \leq 1$

$x^2 - 3x + 2 \leq 0$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x_{1,2} = \frac{3 \pm \sqrt{9 - 8}}{2}$$

$$\begin{cases} x_1 = 2 \\ x_2 = 1 \end{cases}$$



$x \in R \ \wedge \ 1 \leq x \leq 2 \ \Rightarrow \ 1 \leq x \leq 2$

$$\frac{x+2}{x+3} > \frac{2x+3}{x+6}$$

$$\frac{x+2}{x+3} - \frac{2x+3}{x+6} > 0 \Rightarrow \frac{(x+2)(x+6) - (x+3)(2x+3)}{(x+3)(x+6)} > 0$$

$$\Rightarrow \frac{-x^2 - x + 3}{(x+3)(x+6)} > 0$$

$$-x^2 - x + 3 = 0 \quad x_1 = \frac{-1-\sqrt{13}}{2} \quad x_2 = \frac{-1+\sqrt{13}}{2}$$

$$(x+3)(x+6) = 0 \qquad x = -3 \quad x = -6$$

	$(-\infty, -6)$	$[-6, -3)$	$[-3, \frac{-1-\sqrt{13}}{2})$	$[\frac{-1-\sqrt{13}}{2}, \frac{-1+\sqrt{13}}{2})$	$[\frac{-1+\sqrt{13}}{2}, \infty)$
$-x^2 - x + 3$	$-$	$-$	$-$	0 $+$ 0	$-$
$(x+3)(x+6)$	$+$	0 $-$ 0	$+$	$+$	$+$
$\frac{-x^2 - x + 3}{(x+3)(x+6)}$	$-$	∞ $+$ ∞	$-$	0 $+$	0 $-$

$$(-6, -3) \cup (\frac{-1-\sqrt{13}}{2}, \frac{-1+\sqrt{13}}{2})$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|5| = 5$$

$$|-100| = -(-100) = 100$$

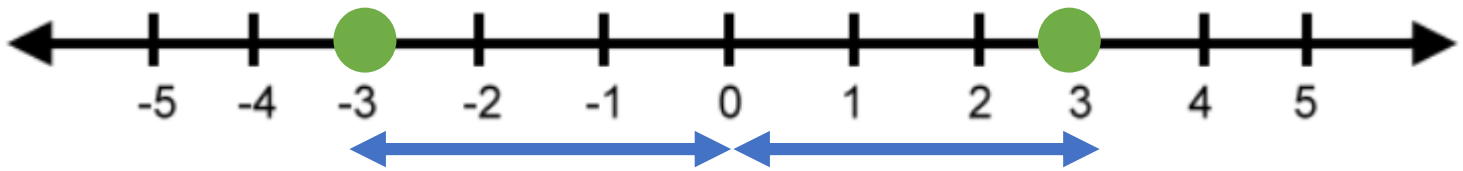
$$|x-2| = \begin{cases} x-2 & \text{if } x-2 \geq 0 \\ -(x-2) & \text{if } x-2 < 0 \end{cases} = \begin{cases} x-2 & \text{if } x \geq 2 \\ -x+2 & \text{if } x < 2 \end{cases}$$

$x - 2 = 0$

$x = 2$

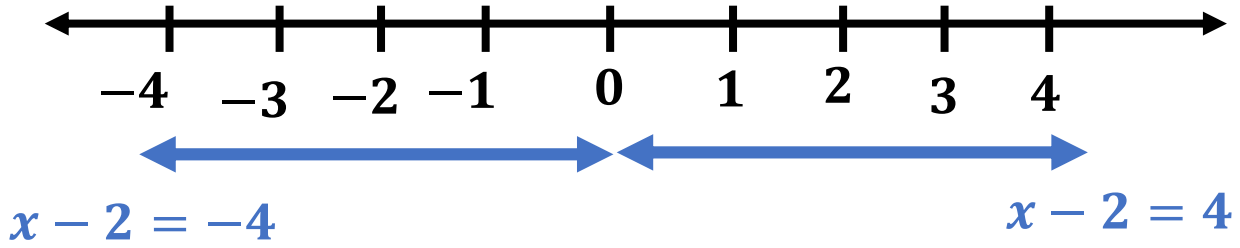
$$|x+3| = \quad \bullet\bullet\bullet \quad = \begin{cases} x+3 & \text{if } x \geq -3 \\ -x-3 & \text{if } x < -3 \end{cases}$$

$$|x| = 3 \quad \Leftrightarrow \quad |x - 0| = 3$$



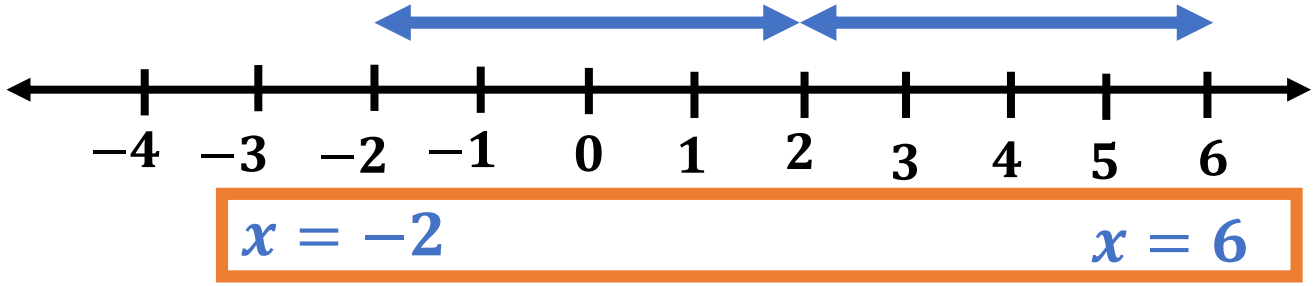
$$|x| = 3 \quad \Rightarrow \quad \begin{matrix} x = 3 \\ x = -3 \end{matrix}$$

$|x-2|=4 \quad \Leftrightarrow \quad |(x-2)-0|=4$



$|x-2|=4 \Rightarrow \begin{matrix} x-2=4 \\ x-2=-4 \end{matrix} \Rightarrow \boxed{\begin{matrix} x=6 \\ x=-2 \end{matrix}}$

$|x-2|=4$



$$\big||x-3|-2\big|=1 \quad \Leftrightarrow \begin{cases} |x-3|-2=1 \\ \text{V} \\ |x-3|-2=-1 \end{cases}$$

$$|x-3|-2=1 \Rightarrow |x-3|=3 \Rightarrow \begin{cases} x-3=3 \rightarrow x=6 \\ x-3=-3 \rightarrow x=0 \end{cases}$$

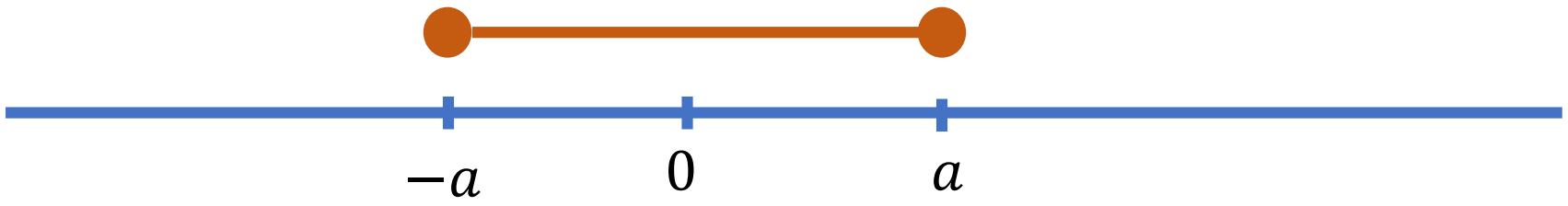
$$|x-3|-2=-1 \Rightarrow |x-3|=1 \Rightarrow \begin{cases} x-3=1 \rightarrow x=4 \\ x-3=-1 \rightarrow x=2 \end{cases}$$

$$x \in \{0,2,4,6\}$$

$$\big||x-2|+1\big| \leq 5$$

Pamatování

$$|x| \leq a$$



$$-a \leq x \leq a$$

$$\big||x-2|+1\big| \le 5 \quad \Leftrightarrow \quad |x-2|+1 \le 5 \quad \Leftrightarrow |x-2| \le 4$$

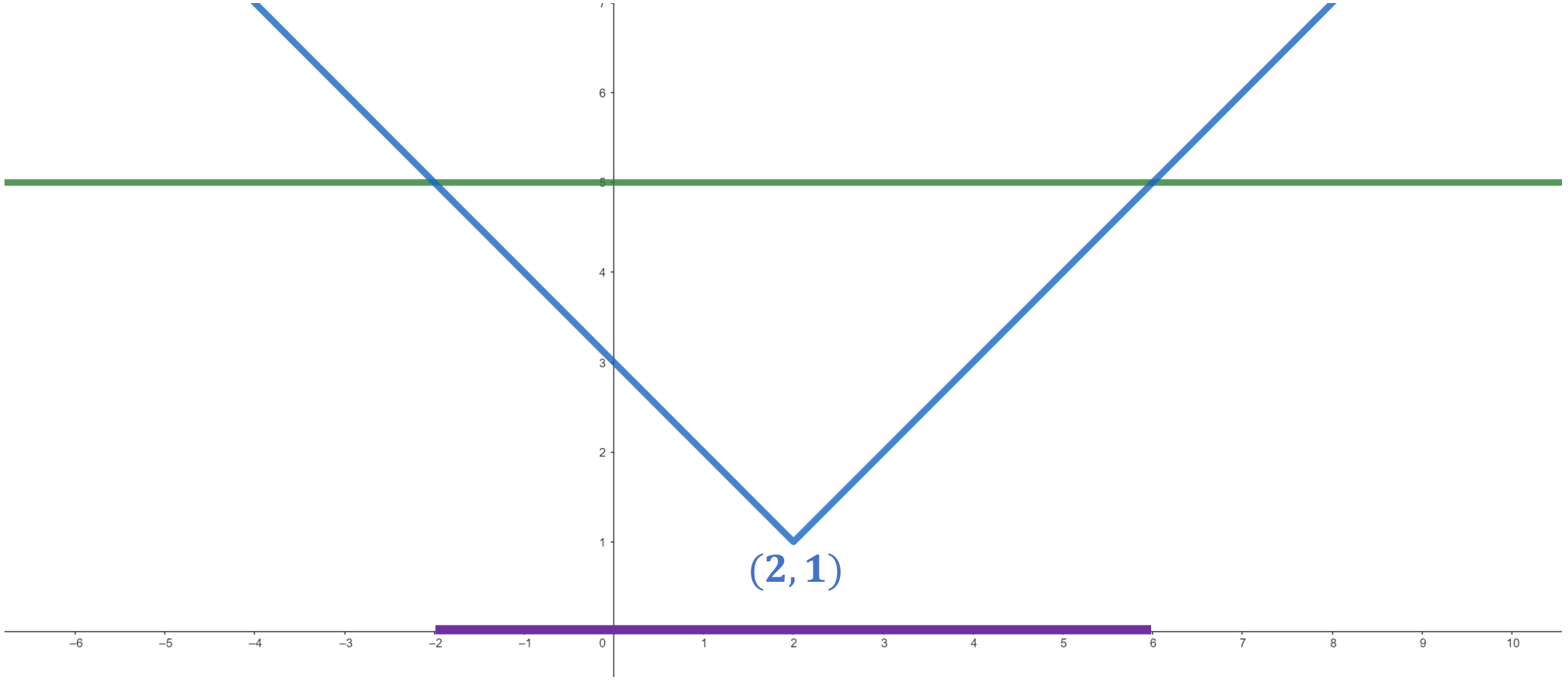
$$\Leftrightarrow -4 \le x-2 \le 4 \qquad \Leftrightarrow -2 \le x \le 6$$

$$\big||x-2|+1\big| = |x-2|+1$$

$$> 0$$

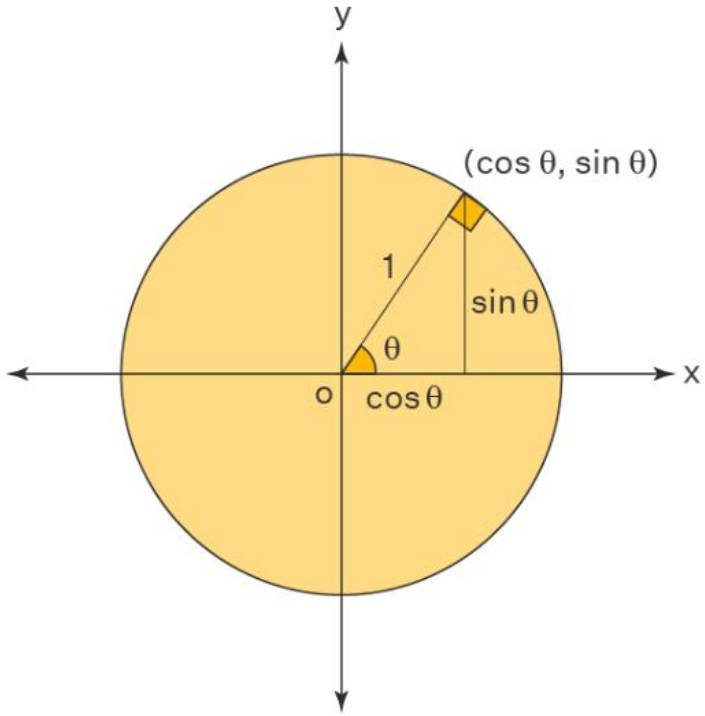
$$\big||x-2|+1\big| \leq 5$$

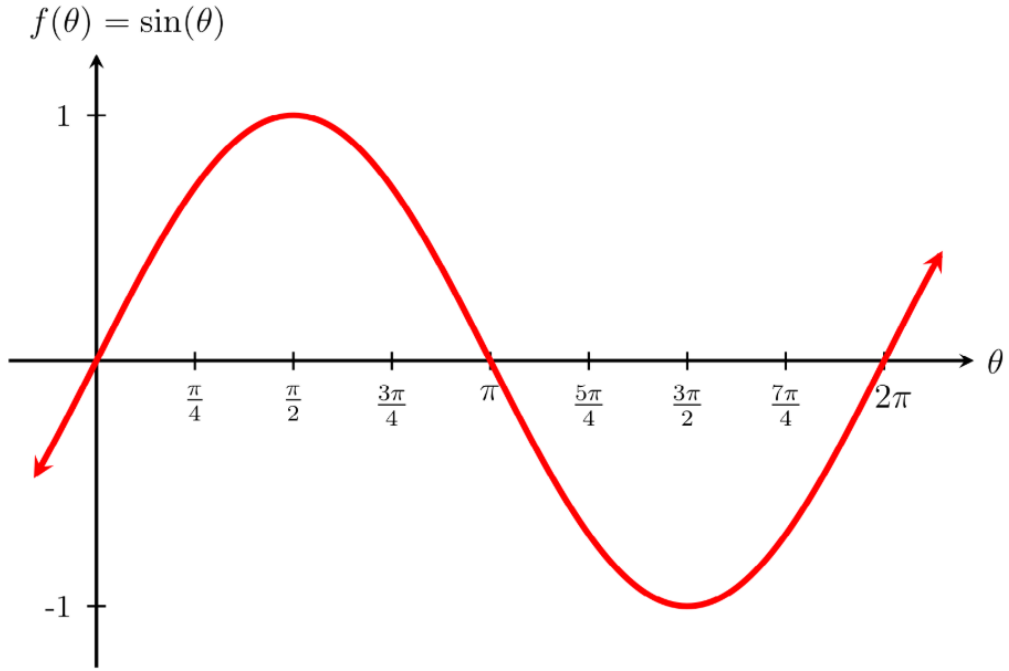
$$f(x) = \big||x-2|+1\big| = |x-2|+1 \qquad g(x) = 5$$



$$x \in [-2, 6]$$

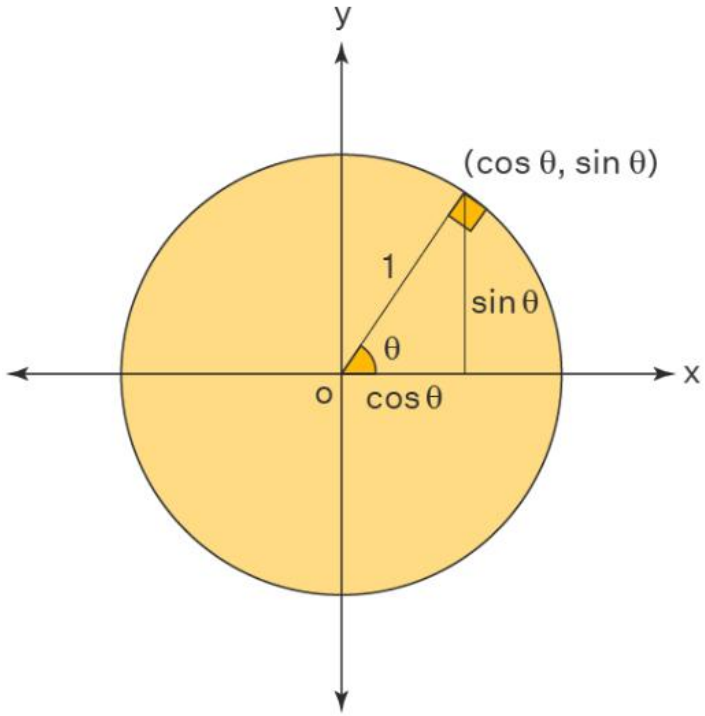
$$f(x) = \sin x$$

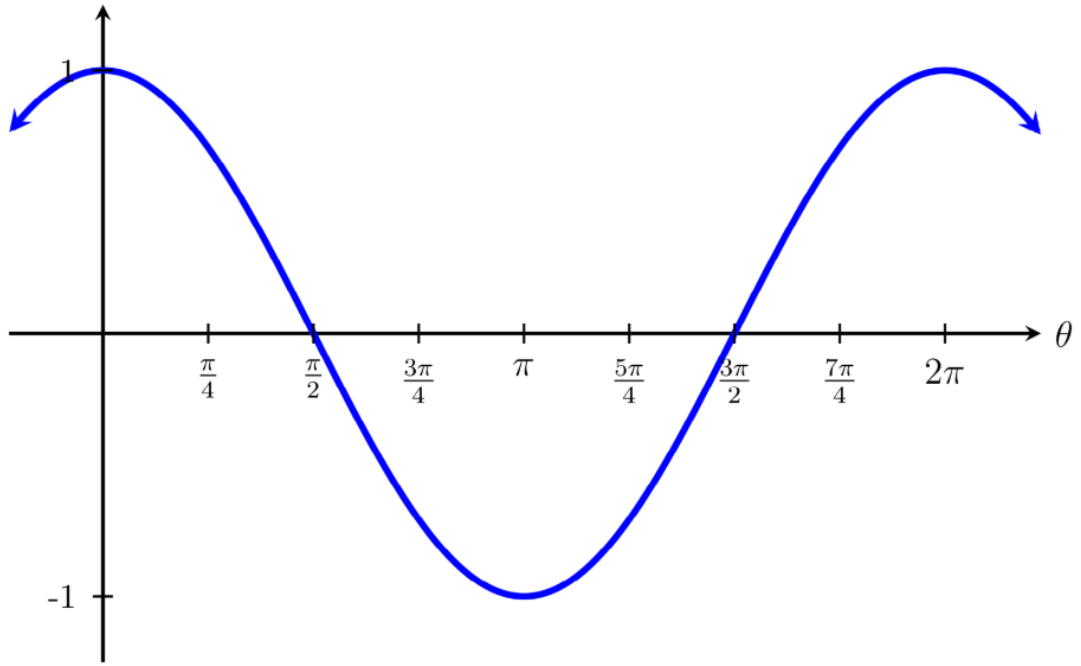




$$D_f = \mathbb{R} \qquad H_f = [-1, 1]$$

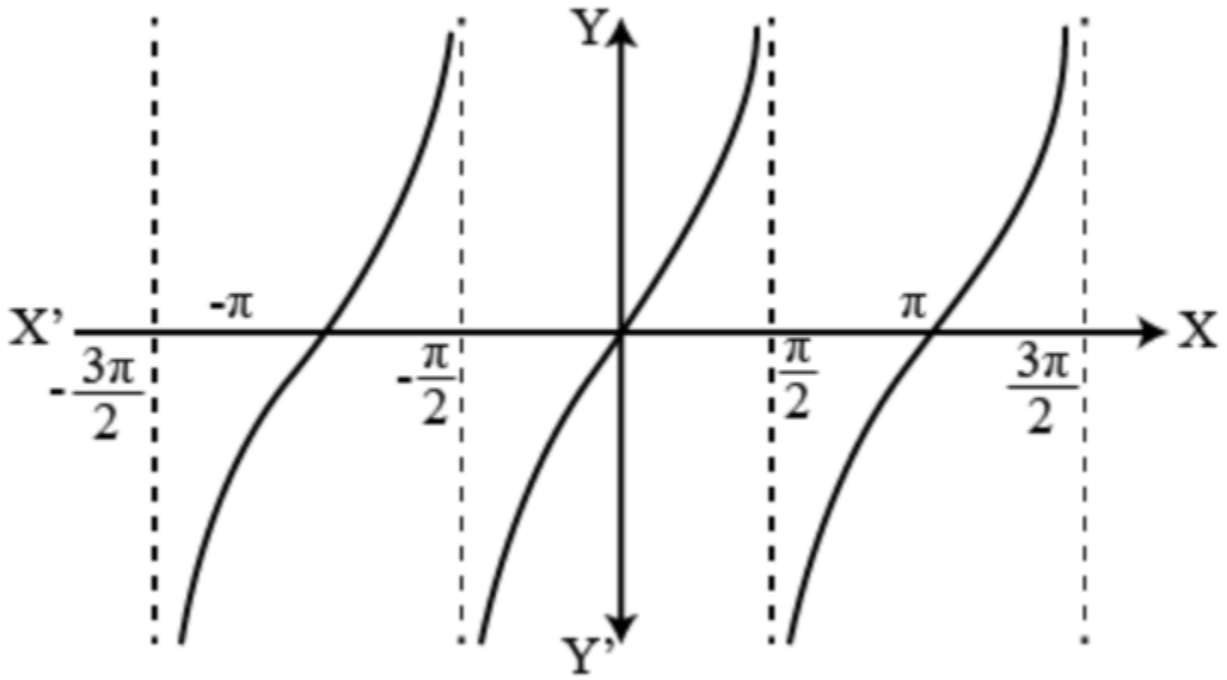
$$f(x) = \cos x$$





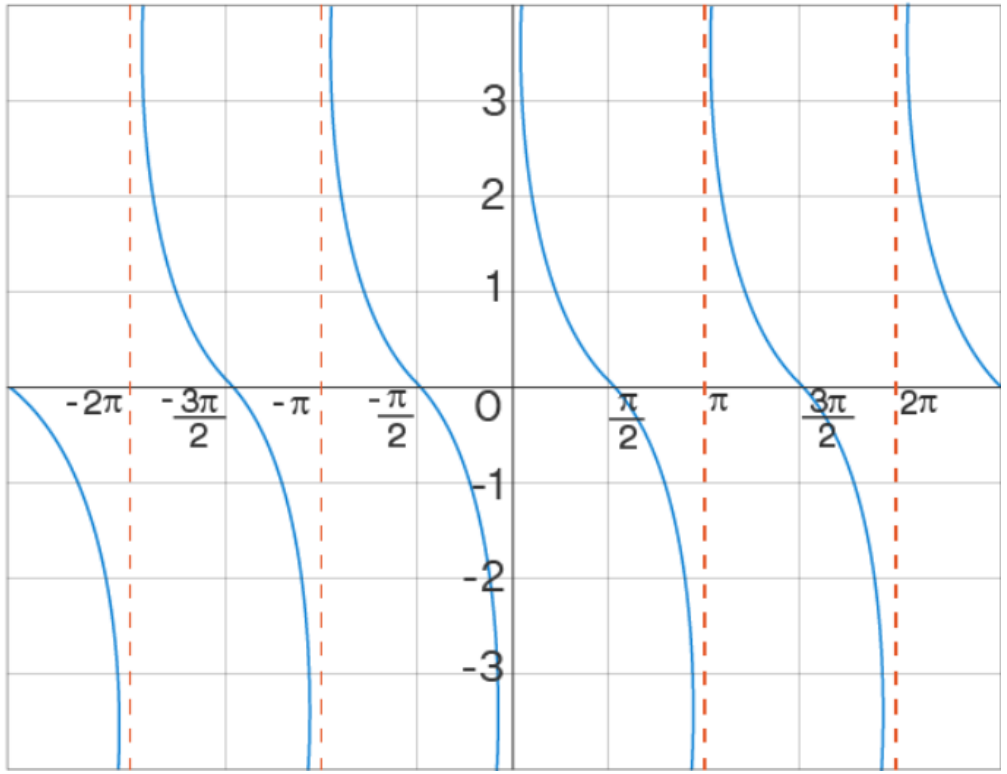
$$D_f = \mathbb{R} \qquad H_f = [-1, 1]$$

$$f(x) = tg\, x$$



$$D_f = \mathbb{R} - \{(2k+1)\frac{\pi}{2} | k \in \mathbb{Z}\} \qquad H_f = \mathbb{R}$$

$$f(x) = ctg\, x$$



$$D_f = \mathbb{R} - \{k\pi | k \in \mathbb{Z}\} \qquad H_f = \mathbb{R}$$

rad	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
degree	0°	30°	45°	60°	90°	180°	270°	360°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	-	0	-	0
cot	-	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-	0	-