Vyšetřete průběh funkce zadané předpisem

$$f(x) = |x-2| - 2\operatorname{arctg} x.$$

$$f(x) = \operatorname{arctg}\left(\frac{x+2}{x}\right) \quad \text{pro } x \neq 0 \quad \text{a} \quad f(0) = \frac{\pi}{2}.$$

$$f(x) = \sqrt[3]{x^3 - x^2 - 2x}.$$

$$f(x) = 2\cos^2 x + \sin^2(2x).$$

$$f(x) = \frac{x}{\ln x}.$$

$$f(x) = (x^2 + 3x + 2) \cdot e^{|x-3|+3}.$$

4. $f(x) = |x-2| - 2\operatorname{arctg} x$ ... 1 $D_f = \mathbb{R}$, spojitá.

1 • symetrie (sudost / lichost / periodicita) nejsou.

• $\lim_{x\to\infty} f(x) = \lim((x-2) - 2\operatorname{arctg} x) = \infty - 2 - \pi = \infty$

1 $\lim_{x\to-\infty} f(x) = \ldots = -\infty$

• asymptoty: $\underline{\text{v } +\infty:}$ $a := \lim_{x\to\infty} \frac{f(x)}{x}$ = zřejmě $= 1$

3 $b := \lim_{x\to\infty} (f(x) - ax) = \lim_{x\to\infty} \left((x-2) - 2\operatorname{arctg} x - 1\cdot x\right) =$

$= \lim(-2 - 2\operatorname{arctg} x) = -2 - \pi$

$\Rightarrow$ asymptota v $+\infty$ je $y = ax + b = x - 2 - \pi$.

$\underline{\text{v } -\infty:}$ $a := \lim_{x\to-\infty} \frac{f(x)}{x} = \lim\left((2-x) - 2\operatorname{arctg} x\right)\cdot\frac{1}{x} = \ldots = -1$

$b := \lim_{x\to-\infty} (f(x) - ax) = \lim_{x\to-\infty}\left((2-x) - 2\operatorname{arctg} x + x\right) = 2 + \pi$

• $\underline{\text{derivace}}$ $f'(x) = \operatorname{sgn}(x-2) - \frac{2}{1+x^2} =$ pro $x \neq 2$

2 $= \begin{cases} x>2: & \frac{x^2-1}{x^2+1} > 0 \iff x^2-1>0 \iff x \in (-\infty,-1)\cup(1,\infty) \\ x<2: & \frac{-x^2-3}{x^2+1} > 0 \iff -x^2-3>0 \iff +x^2+3<0 \iff x \in \emptyset. \end{cases}$

2

Tabulka:

	$(-\infty,2)$	$(2,\infty)$
f	$\searrow$ ~~$(-\infty,2)$~~	$\nearrow$
f'	$\ominus$	$\oplus$

$\Rightarrow$ globální (i lokální) $\underline{\text{minimum}}$

$f(2) = -2\operatorname{arctg} 2$

3

• $\underline{\text{2. der:}}$ $f''(x) = \left(\operatorname{sgn}(x-2) - \frac{2}{1+x^2}\right)' =$

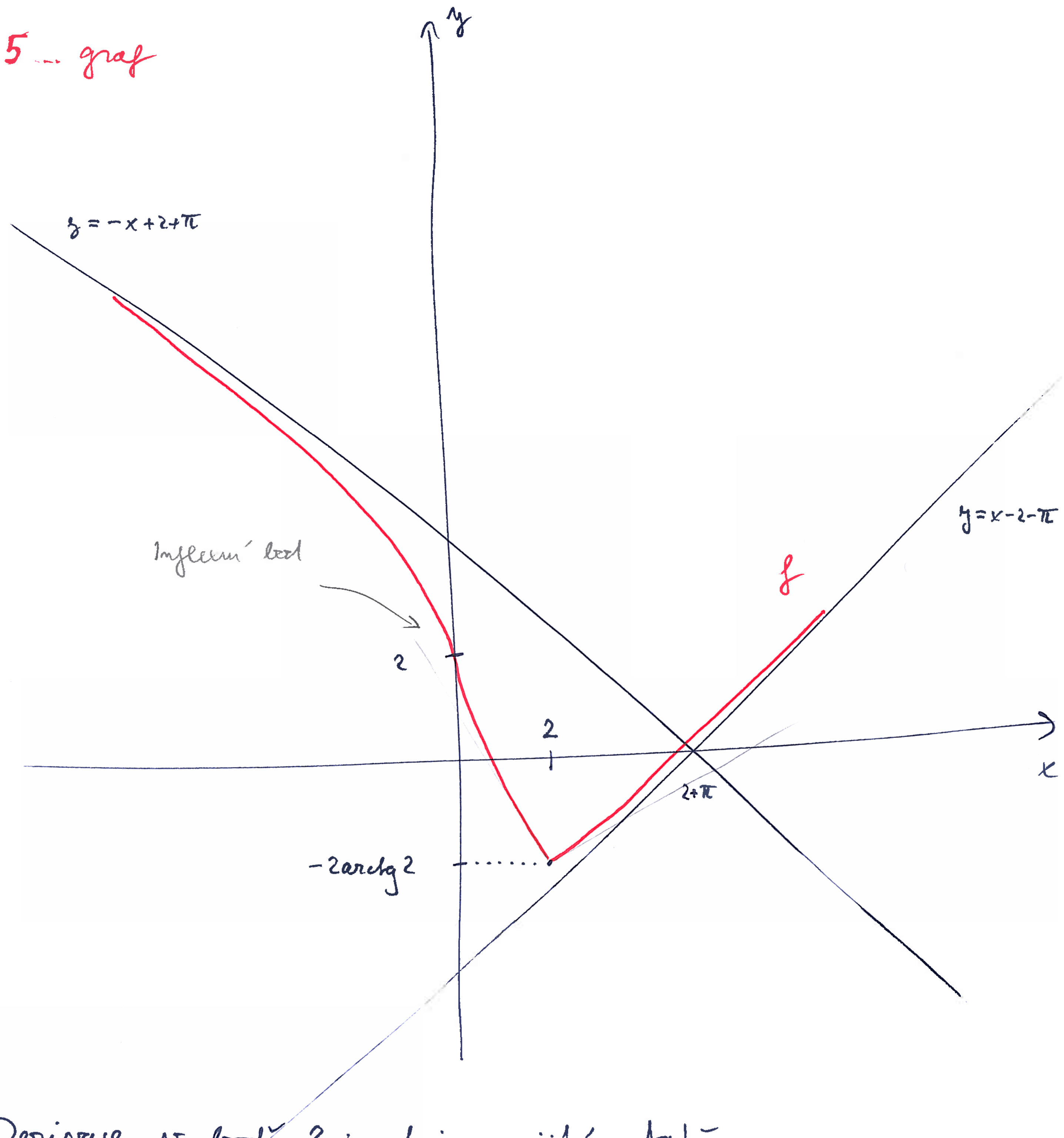
2

$= 0 - \left(\frac{-2\cdot 2x}{(1+x^2)^2}\right) = \frac{4x}{(1+x^2)^2} \Rightarrow$

1

	$(-\infty,0)$	$(0,2)$	$(2,\infty)$
f''	$\ominus$	$\oplus$	$\oplus$
f	$\cap$	$\cup$	$\cup$

5 ... graf



• <u>Derivace v bodě 2:</u> f je spojitá, takže

2 $f'_+(2) = \lim\limits_{x \to 2+} f'(x)$, má-li P.S.S. a podobně zleva.

<u>Tedy:</u> $f'_+(2) = \lim\limits_{x \to 2+} \frac{x^2-1}{x^2+1} = \frac{4-1}{4+1} = \frac{3}{5}$

$f'_-(2) = \lim\limits_{x \to 2-} \frac{-x^2-3}{x^2+1} = \frac{-4-3}{4+1} = \frac{-7}{5}$

4.| $f(x) = \operatorname{arctg}\left(\frac{x+2}{x}\right)$ pro $x \neq 0$, $f(0) := \frac{\pi}{2}$.

Podle def. je $D_f = \mathbb{R}$, spojitá ve všech b. $\mathbb{R} \setminus \{0\}$.

$$\lim_{x \to 0_+} f(x) = \lim_{x \to 0_+} \operatorname{arctg}\Big(\underbrace{\frac{x+2}{x}}_{\text{"}\frac{2}{0_+}\text{"} = +\infty}\Big) = \text{"}\operatorname{arctg} \infty\text{"} = \frac{\pi}{2}.$$

2b

$$\lim_{x \to 0_-} f(x) = \ldots = \text{"}\operatorname{arctg}(-\infty)\text{"} = -\frac{\pi}{2}.$$

Tedy: $\lim\limits_{x \to 0_+} f(x) = f(0) \neq \lim\limits_{x \to 0_-} f(x)$, takže

1b

f je v nule spojitá zprava, ale ne zleva.

1b $\lim\limits_{x \to \pm\infty} \operatorname{arctg}\left(\frac{x+2}{x}\right) = \operatorname{arctg} 1 = \frac{\pi}{4}$. $\Rightarrow$ Přímka $A(x) = 0 \cdot x + \frac{\pi}{4}$ je asymptota v $\pm\infty$.

Funkce není sudá ani lichá, není periodická.

$$f'(x) = \frac{1}{1 + \left(\frac{x+2}{x}\right)^2} \cdot \frac{1 \cdot x - (x+2) \cdot 1}{x^2} = \frac{1}{\frac{x^2 + x^2 + 4x + 4}{x^2}} \cdot \frac{-2}{x^2} =$$

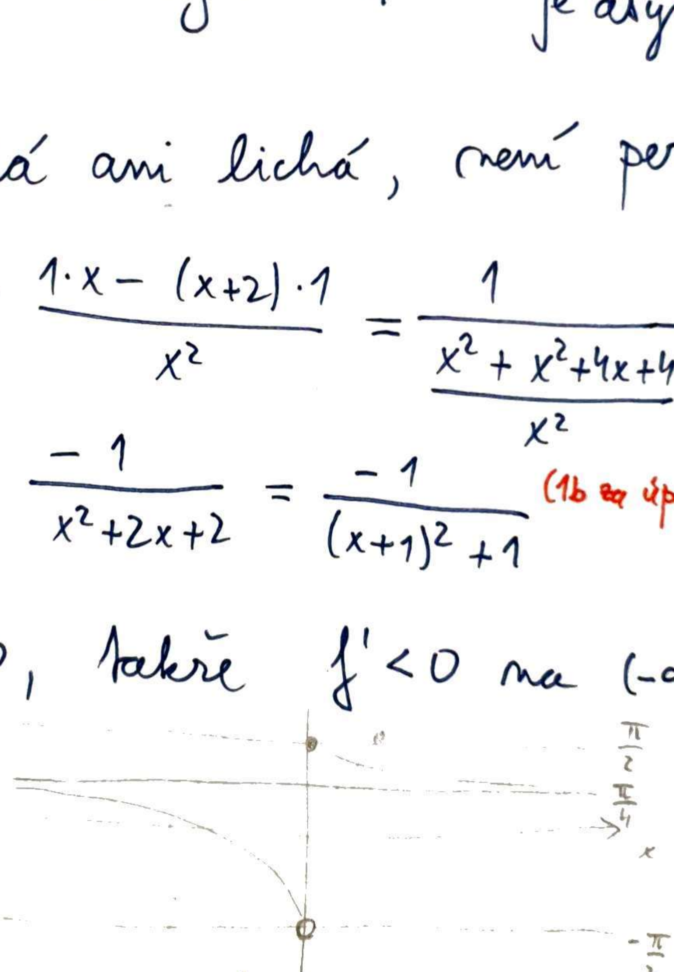
3b

$$= \frac{-2}{2x^2 + 4x + 4} = \frac{-1}{x^2 + 2x + 2} = \frac{-1}{(x+1)^2 + 1} \text{ (1b za úpravu) }, \; x \in \mathbb{R} \setminus \{0\}.$$

Jmenovatel vždy > 0, takže $f' < 0$ na $(-\infty, 0)$ i $(0, \infty)$:

	$(-\infty,0)$	$(0,\infty)$
f'	$\ominus$	$\ominus$
f	$\searrow$	$\searrow$

2b



Tedy v bodě 0 je globální (i lokální) ostré maximum, minimum zřejmě není.

Derivace v bodě 1: zřejmě $f'_-(1) = \infty$.

Díky spojitosti v 0 zprava a V. o lim. derivace:

2b

$$f'_+(0) \underset{\substack{\uparrow\\ \text{má-li PSS}}}{=} \lim_{x \to 0_+} f'(x) = \lim_{x \to 0_+} \frac{-1}{(x+1)^2+1} = \frac{-1}{(0+1)^2+1} = -\frac{1}{2}.$$

Pro zajímavost: $\lim\limits_{x \to 0_-} f'(x) = \ldots = -\frac{1}{2}$, ale $\neq f'_-(0)$.

$$f''(x) = \left(\frac{-1}{(x+1)^2+1}\right)' = -\frac{-1}{((x+1)^2+1)^2} \cdot 2(x+1) = \frac{2(x+1)}{\underbrace{((x+1)^2+1)^2}_{>0,\ x \in \mathbb{R}}}$$

4b

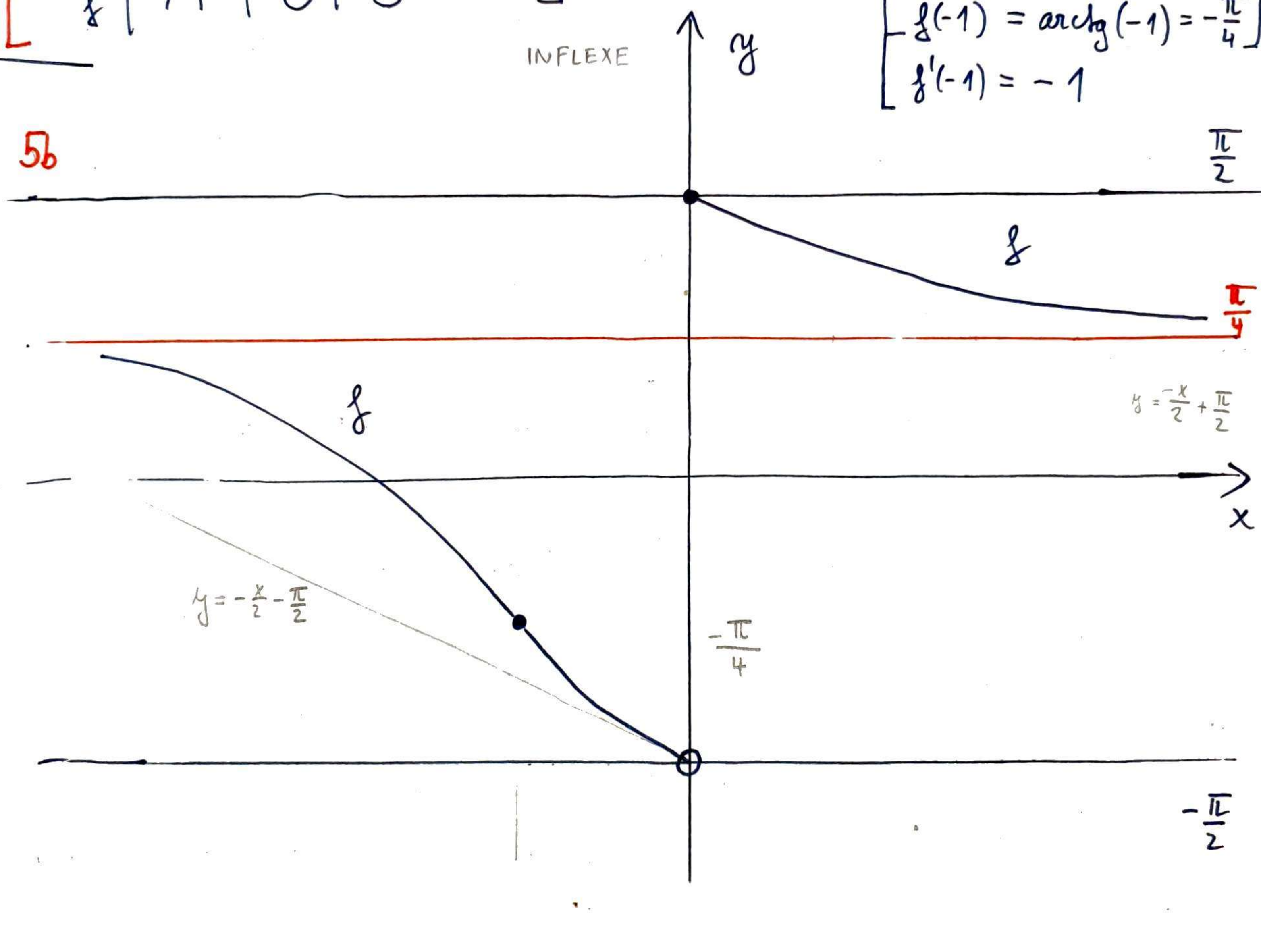
	$(-\infty,-1)$	$(-1,0)$	$(0,\infty)$
f''	$\ominus$	$\oplus$	$\oplus$
f	$\cap$	$\cup$	$\cup$

$\Rightarrow$ Bod -1 je inflexní bod f.

$f(-1) = \operatorname{arctg}(-1) = -\frac{\pi}{4}$

$f'(-1) = -1$

5b



(4.) $f(x) = \sqrt[3]{x^3 - x^2 - 2x}$ ... 1b • $D_f = \mathbb{R}$, f je spoj. na $\mathbb{R}$

1b • symetrie nejsou • $\lim_{x\to\pm\infty} f(x) = \pm\infty$

3b • asymptoty: $a := \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \sqrt[3]{1 - \frac{1}{x} - \frac{2}{x^2}} = 1$

$$b := \lim_{x\to\infty} (f(x) - \underbrace{x}_{a\cdot x = x}) = \lim_{x\to\infty} \left(\sqrt[3]{x^3-x^2-2x} - x\right) =$$

$$= \lim_{x\to\infty} \frac{x^3 - x^2 - 2x - x^3}{(x^3-x^2-2x)^{2/3} + (x^3-x^2-2x)^{1/3}x + x^2} = \lim_{x\to\infty} \frac{-x^2-2x}{x^2\left((1-\dots)^{2/3} + (1-\dots)^{1/3}\cdot 1 + 1\right)}$$

$$= \lim \frac{1}{\underbrace{(\,)}_{\to 1} + \underbrace{(\,)}_{\to 1} + 1} \cdot \lim_{x\to\infty} \frac{-x^2-2x}{x^2} = \frac{1}{3} \cdot \lim \frac{-1-\frac{2}{x}}{1} = -\frac{1}{3}$$

Výpočet projde zcela stejně i pro $x \to -\infty$. Tedy

$A(x) = ax + b = x - \frac{1}{3}$ je asymptota v $+\infty$ i $-\infty$.

2b $$f'(x) = \left((x^3 - x^2 - 2x)^{\frac{1}{3}}\right)' = \frac{1}{3}(x^3-x^2-2x)^{-\frac{2}{3}} \cdot (3x^2 - 2x - 2)$$

kořeny polynomů: • $x^3 - x^2 - 2x = x\cdot(x^2 - x - 2) = x\cdot(x-2)(x+1)$

• $3x^2 - 2x - 2 = 0$ ... $x_{1,2} = \frac{2 \pm \sqrt{4 + 24}}{6} = \frac{2 \pm 2\sqrt{7}}{6} = \left\langle \begin{array}{l} \frac{1+\sqrt{7}}{3} \\ \frac{1-\sqrt{7}}{3} \end{array}\right.$

Tedy: 1b • vzorec pro $f'(x)$ platí pro $x \in \mathbb{R}\setminus\{-1, 0, 2\}$.

• znaménko $f'(x)$ nezávisí na $\frac{1}{3}(x^3 - x^2 - 2x)^{-\frac{2}{3}}$ (vždy kladné)

tj. $f'(x) > 0 \iff (3x^2 - 2x - 2) > 0 \iff x \in (-\infty, \frac{1-\sqrt{7}}{3}) \vee x \in (\frac{1+\sqrt{7}}{3}, \infty)$.

2b

Celkem:

	$(-\infty, \frac{1-\sqrt{7}}{3})$	$(\frac{1-\sqrt{7}}{3}, \frac{1+\sqrt{7}}{3})$	$(\frac{1+\sqrt{7}}{3}, \infty)$
f'	$\oplus$	$\ominus$	$\oplus$
f	$\nearrow$	$\searrow$	$\nearrow$

$-1 < \frac{1-\sqrt{7}}{3} < 0 < \frac{1+\sqrt{7}}{3} < 2$

V bodě $\frac{1-\sqrt{7}}{3}$ je tedy lokální max. f,

1b

$\frac{1+\sqrt{7}}{3}$ min f.

$$f\left(\frac{1-\sqrt{7}}{3}\right) = \sqrt[3]{\left(\frac{1-\sqrt{7}}{3}\right)^3 - \left(\frac{1-\sqrt{7}}{3}\right)^2 - 2\left(\frac{1-\sqrt{7}}{3}\right)} = \ldots$$

Derivace v bodech $-1, 0, 2$: f je v těchto b. spoj.

2b

(dokonce na $\mathbb{R}$), takže užijeme větu o lim. derivace:

- $$\lim_{x\to -1} f'(x) = \lim_{x\to -1} \frac{1}{3}\left(x^3 - x^2 - 2x\right)^{-\frac{2}{3}}\left(3x^2 - 2x - 2\right) =$$

$$= \frac{1}{3}\left(3(-1)^2 - 2(-1) - 2\right) \cdot \lim_{x\to -1} \frac{1}{\left(\sqrt[3]{x^3 - x^2 - 2x}\right)^2} = \text{"}\frac{3}{3} \cdot \frac{1}{0_+}\text{"} = \infty$$

- $\lim_{x\to 0} f'(x) =$ "podobně" $= -\infty$
- $f'(2) = \lim_{x\to 2} f'(x) = \ldots = \infty$.

Celkem: $f'(-1) = \infty$

$f'(0) = -\infty$

$f'(2) = \infty$.

---

2. DERIVACE „NEBYLA POTŘEBA" - pouze 2b

2b

$$f''(x) = \left(\frac{1}{3}\left(x^3 - x^2 - 2x\right)^{-\frac{2}{3}}\left(3x^2 - 2x - 2\right)\right)' =$$

$$= \frac{1}{3} \cdot \frac{-2}{3} \cdot \left(x^3 - x^2 - 2x\right)^{-\frac{5}{3}}\left(3x^2 - 2x - 2\right)^2 +$$

$$+ \frac{1}{3} \cdot \left(x^3 - x^2 - 2x\right)^{-\frac{2}{3}}\left(6x - 2\right) =$$

$$= \frac{1}{3} \cdot \left(x^3 - x^2 - 2x\right)^{-2/3} \cdot \left(-\frac{2}{3} \cdot \frac{\left(3x^2 - 2x - 2\right)^2}{x^3 - x^2 - 2x} + (6x - 2)\right) =$$

$$= \frac{1}{3}(\ldots)^{-2/3} \cdot \frac{1}{\ldots}\left(-\frac{2}{3}\left(9x^4 + 4x^2 + 4 - 12x^3 - 12x^2 + 8x\right) + \cancel{6x^4} - 6x^3 - \cancel{12x^2}\right.$$

$$\left. - 2x^3 + 2x^2 + 4x\right) = \frac{1}{3}(\ldots)^{-2/3} \cdot \frac{1}{\ldots}\left(\cancel{2x^3} + \left(\frac{2}{3}\cdot 8 - 10\right)x^2 + \left(-\frac{2}{3}\cdot 8 + 4\right)x - \cancel{2x^3}\right.$$

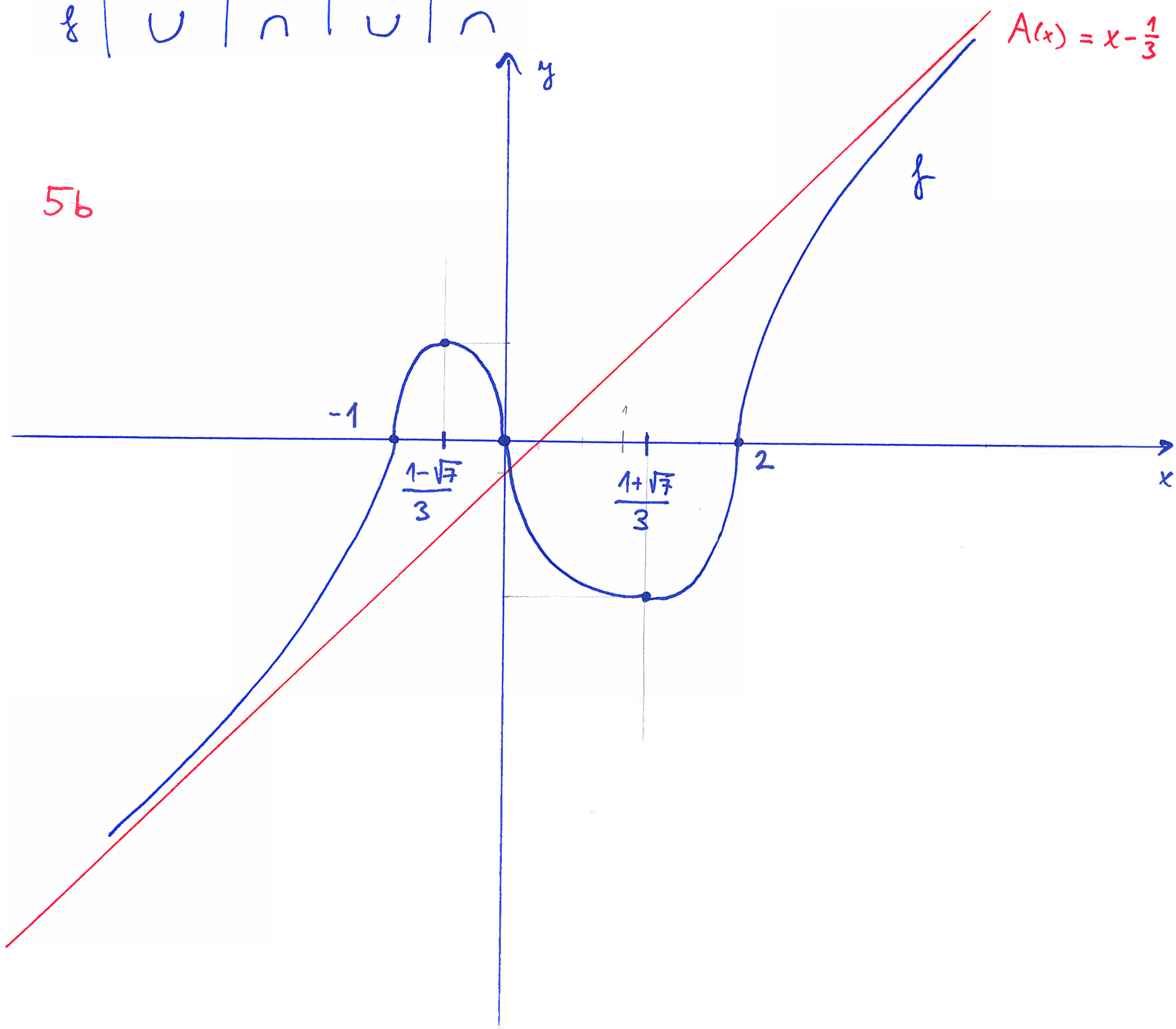
$$\left. - \frac{8}{3}\right) = \frac{1}{3} \cdot \frac{(\ldots)^{-2/3}}{(\ldots)} \cdot \left(\frac{-14}{3}x^2 - \frac{4}{3}x - \frac{8}{3}\right) = \frac{-2}{9} \cdot (\ldots)^{-5/3}\left(+7x^2 + 2x + 4\right)$$

$$f''(x) = -\frac{2(7x^2+2x+4)}{9\cdot(x^3-x^2-2x)^{5/3}}.$$

Polynom v čitateli je kladný pro každé $x \in \mathbb{R}$ (neboť $D = 2^2 - 4\cdot 7\cdot 4 < 0$), takže znaménko f'' ovlivňuje pouze $(x^3-x^2-2x) = x\cdot(x-2)(x+1)$. Celkem:

	$(-\infty,-1)$	$(-1,0)$	$(0,2)$	$(2,\infty)$
f''	$\oplus$	$\ominus$	$\oplus$	$\ominus$
f	$\cup$	$\cap$	$\cup$	$\cap$

$-1, 0, 2$ jsou inflexní body f.



(4.) $2\cos^2 x + \sin^2(2x) = f(x)$

- $D_f = \mathbb{R}$, 1b spojitá na $\mathbb{R}$, sudá 1b, $\lim_{x\to\pm\infty} f(x)$ neexistují
- $f(x+\pi) = 2\cos^2(x+\pi) + \sin^2(2x+2\pi) = 2(-\cos x)^2 + \sin^2 2x =$

$$= 2\cos^2 x + \sin^2 2x = f(x).$$

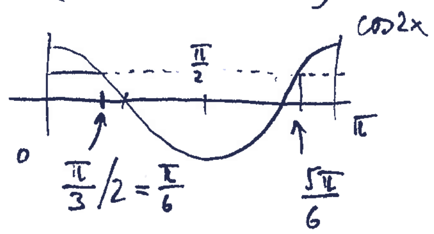
2b Tedy f je π-periodická $\Rightarrow$ stačí vyšetřit na $[0,\pi]$
(2π-per $\Rightarrow$ 1b.)
(dokonce, díky sudosti, stačí na $[0,\frac{\pi}{2}]$) (resp. $[0,\pi)$)

- $f'(x) = \left(2\cos^2 x + \sin^2(2x)\right)' = 2\cdot 2\cdot \cos x\,(-\sin x) + 2\sin(2x)\cos(2x)\cdot 2$

2b $= 4\sin 2x\cdot\cos 2x - 4\sin x\cos x =$

$= 4\sin 2x\cdot \cos 2x - 2\sin 2x = 2\sin 2x\,(2\cos 2x - 1)$

- $\sin 2x = 0$, $x\in[0,\pi)$: $x\in\{0,\frac{\pi}{2}\}$
- $\cos 2x = \frac{1}{2}$, $x\in[0,\pi)$: $x\in\{\frac{\pi}{6},\frac{5\pi}{6}\}$



4b

	$[0,\frac{\pi}{6}]$	$[\frac{\pi}{6},\frac{\pi}{2}]$	$[\frac{\pi}{2},\frac{5\pi}{6}]$	$[\frac{5\pi}{6},\pi]$
f'	+	−	+	−
f	↗	↘	↗	↘

$\Rightarrow$ lok max: $\frac{\pi}{6}$, $\frac{5\pi}{6}$ $(+k\pi,\ k\in\mathbb{Z})$

lok min: 0, $\frac{\pi}{2}$

- $f(\frac{\pi}{6}) = 2\cos^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} = 2\left(\frac{\sqrt3}{2}\right)^2 + \left(\frac{\sqrt3}{2}\right)^2 = \frac{9}{4}$

2b $f(\frac{5\pi}{6}) = f(-\frac{\pi}{6}) = f(\frac{\pi}{6}) = \frac{9}{4}$

- $f(0) = 2\cos^2 0 + \sin^2 0 = 2$
- $f(\frac{\pi}{2}) = 2\cos^2\frac{\pi}{2} + \sin^2\pi = 0$

$H_f = [0,\frac{9}{4}]$

$f''(x) = \left(2\cdot\sin 2x\,(2\cos 2x - 1)\right)' = 2\cdot\big(2\cos 2x\,(2\cos 2x - 1) +$

2b+ $\cancel{2}$ $\sin 2x\cdot 2\cdot 2\cdot(-\sin 2x)\big) =$ [$\cos 2t = \cos^2 t - \sin^2 t$

$= 4\cdot\left(2\cos^2 2x - \cos 2x - 2\sin^2 2x\right) =$

$= 4\cdot\left(2\cos 4x - \cos 2x\right) = 4\left(-\cos 2x + 4\cos^2 2x - 2\right)$

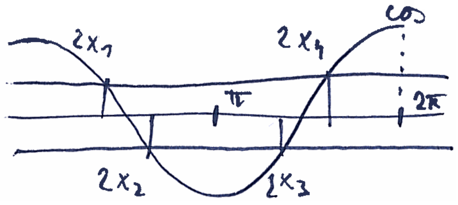
$4\cos^2 2x - \cos 2x - 2 = 0 \rightsquigarrow$ SUBST. $\rightsquigarrow 4a^2 - a - 2 = 0$

$$a_{1,2} = \frac{1 \pm \sqrt{1+32}}{8} = \begin{cases} \frac{1+\sqrt{33}}{8} \\ \frac{1-\sqrt{33}}{8} \end{cases}$$

$$\left[4\left(-(\cos^2 x - \sin^2 x) + 4\cos^2 2x - 2\right) = 4\left(1 - 2\cos^2 x + 4\cos^2 2x - 2\right) = 16\cos^2 2x - 8\cos^2 x - 4 \right]$$

$$\frac{1+\sqrt{33}}{8} \in \left(\frac{1+\sqrt{25}}{8}, \frac{1+\sqrt{36}}{8}\right) = \left(\frac{6}{8}, \frac{7}{8}\right), \quad \frac{1-\sqrt{33}}{8} \in \left(\frac{1-\sqrt{36}}{8}, \frac{1-\sqrt{25}}{8}\right) = \left(\frac{-5}{8}, -\frac{1}{2}\right)$$

$$\cos 2x = \frac{1+\sqrt{33}}{8}$$



1b $2x_1 = \arccos \frac{1+\sqrt{33}}{8}$

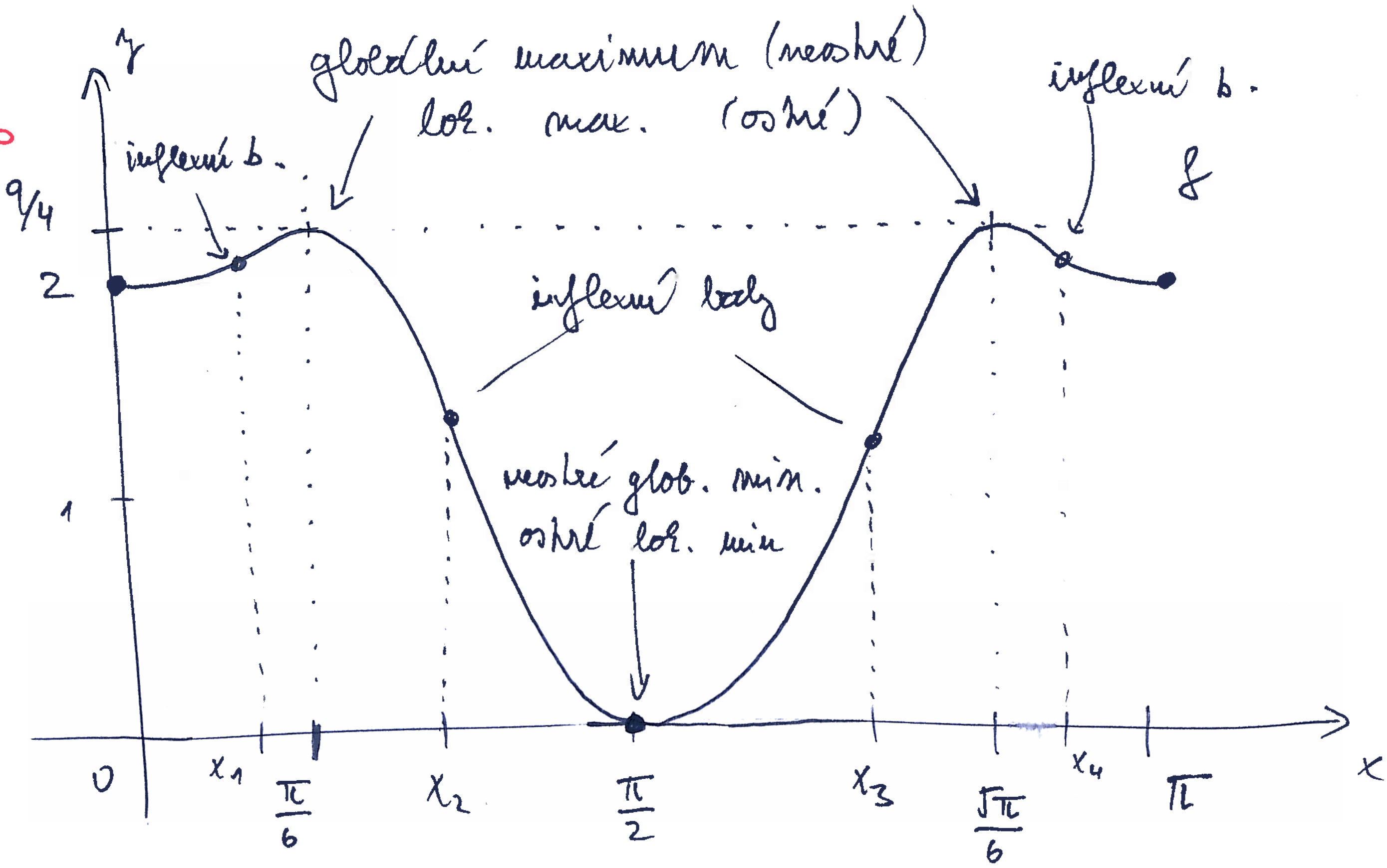
$2x_4 = 2\pi - \arccos \frac{1+\sqrt{33}}{8}$

$2x_2 = \arccos \frac{1-\sqrt{33}}{8}$

$2x_3 = 2\pi - \arccos \frac{1-\sqrt{33}}{8}$

x_1, x_2, x_3, x_4 inflexní body

5b



④ $f(x) = \dfrac{x}{\ln x}$

2 $D_f = (0,\infty)\setminus\{1\} = \cancel{(0,\infty)}\ (0,1)\cup(1,\infty)$

spojitá na D_f, symetrie nejsou.

$\lim\limits_{x\to 0_+} f(x) = \dfrac{0}{-\infty} = 0$, $\quad \lim\limits_{x\to 1_-} f(x) \text{ "}=\text{" } \dfrac{1}{0_-}\text{"} = -\infty$,

3

$\lim\limits_{x\to\infty} f(x) \overset{l'H}{\underset{\frac{\infty}{\infty}}{=}} \lim\limits_{x\to\infty} \dfrac{1}{\frac{1}{x}} = \infty$, $\quad \lim\limits_{x\to 1_+} f(x) \text{ "}=\text{" } \dfrac{1}{0_+}\text{"} = +\infty$.

Asymptota v ∞: $\quad a = \lim\limits_{x\to\infty} \dfrac{f(x)}{x} = \lim\limits_{x\to\infty} \dfrac{x}{x\cdot\ln x} = 0$

1

$b = \lim\limits_{x\to\infty}\left(f(x) - 0\cdot x\right) = \lim\limits_{x\to\infty} f(x) = \infty \implies$

$\implies$ asymptota není.

2 $f'(x) = \dfrac{1\cdot\ln x - x\cdot\frac{1}{x}}{\ln^2 x} = \dfrac{\ln x - 1}{\underbrace{\ln^2 x}_{>0}} > 0 \iff \ln x > 1$

$\iff x > e$. $\quad (x\in D_f)$

2

	$(0,1)$	$(1,e)$	(e,∞)
f'	$-$	$-$	$+$
f	$\searrow$	$\searrow$	$\nearrow$

V bodě $x = e$ je lokální minimum,

$f(e) = \dfrac{e}{\ln e} = e$

2 $f''(x) = \dfrac{\frac{1}{x}\ln^2 x - (\ln x - 1)\,2\ln x\cdot\frac{1}{x}}{(\ln^2 x)^2} = \dfrac{\ln^2 x - 2\ln^2 x + 2\ln x}{x(\ln^2 x)^2} =$

$= \dfrac{2\ln x - \ln^2 x}{x\ln^4 x} > 0 \iff 2\ln x - \ln^2 x > 0$

$(x\in D_f)$

$\Leftrightarrow \quad \ln x \cdot (2 - \ln x) > 0$

$\Leftrightarrow \quad (\ln x > 0 \ \wedge \ \ln x < 2) \ \vee \ (\ln x < 0 \ \wedge \ \ln x > 2)$

$\Leftrightarrow \quad x \in (1, e^2)$

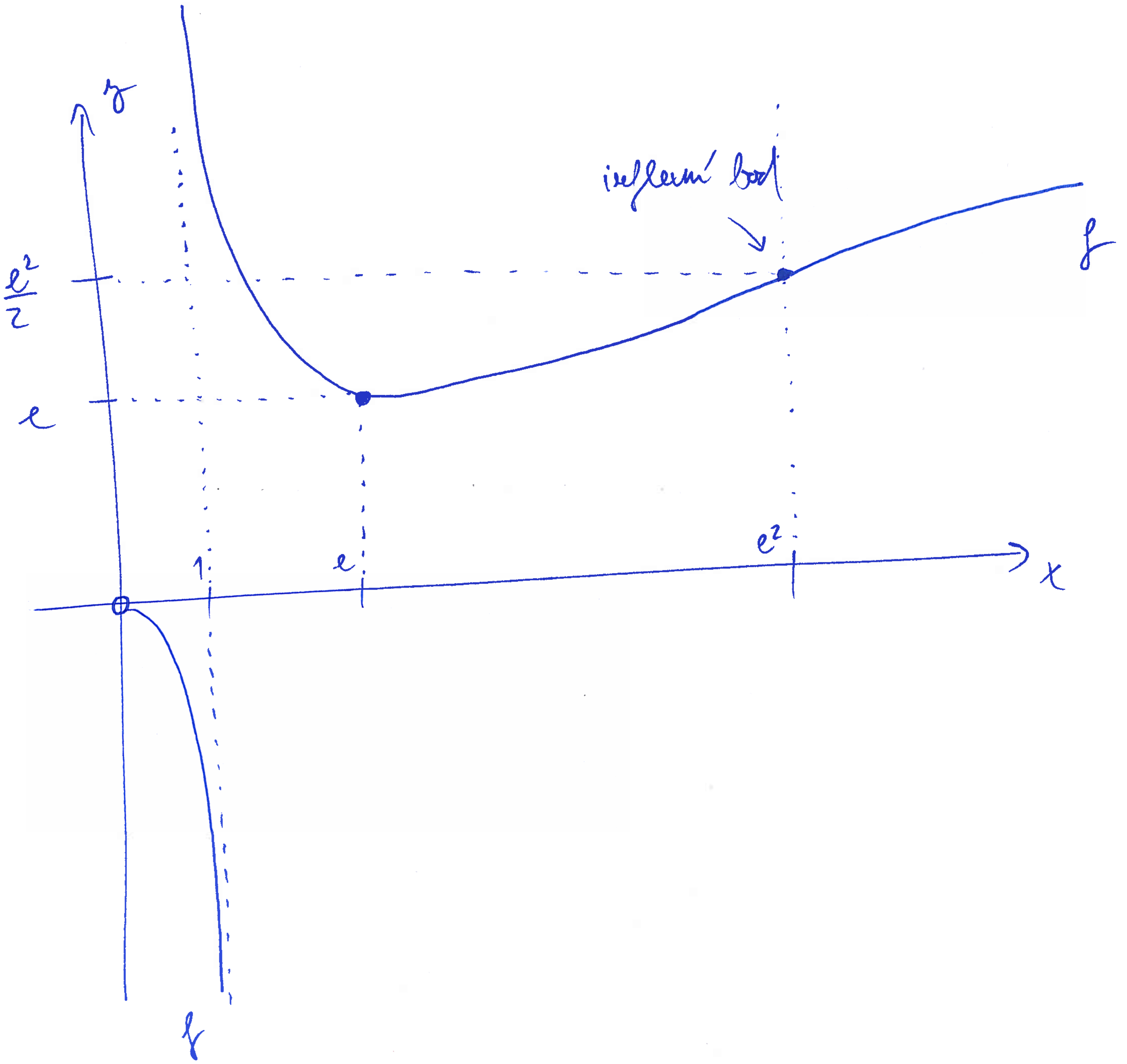
3

	$(0,1)$	$(1,e^2)$	(e^2,∞)
f''	$\ominus$	$\oplus$	$\ominus$
f	$\cap$	$\cup$	$\cap$

e^2 je inflexní bod

$f(e^2) = \frac{e^2}{\ln e^2} = \frac{e^2}{2}$

5



4. $f(x) = (x^2+3x+2)\cdot e^{|x-3|+3}$

1 $D_f = \mathbb{R}$, spojitá na $\mathbb{R}$, symetrie nejsou

1 $\lim_{x\to\pm\infty} f(x) = \infty$, asymptoty nejsou $\left(\lim_{x\to\pm\infty} \frac{f(x)}{x} = \pm\infty\right)$

1 $f'(x) = (2x+3)\cdot e^{|x-3|+3} + (x^2+3x+2)\cdot e^{|x-3|+3}\cdot \operatorname{sgn}(x-3)$

$= e^{|x-3|+3}\left(2x+3+\operatorname{sgn}(x-3)(x^2+3x+2)\right)$, tj.

1 pro $x>3$: $f'(x) = e^{|x-3|+3}(x^2+5x+5) =$

$\frac{-5\pm\sqrt{25-20}}{2} = \begin{cases} \frac{-5+\sqrt{5}}{2} \\ \frac{-5-\sqrt{5}}{2}\end{cases}$ $= \underbrace{e^x}_{>0}(x^2+5x+5) > 0 \iff x^2+5x+5>0$

$\iff x \in \left(-\infty, \frac{-5-\sqrt{5}}{2}\right) \cup \left(\frac{-5+\sqrt{5}}{2}, \infty\right)$

Tedy (protože $\frac{-5+\sqrt{5}}{2} < 0 < 3$) f je rostoucí na $(3,\infty)$

1 pro $x<3$: $f'(x) = e^{-x+3+3}(2x+3-x^2-3x-2) =$

$\frac{-1\pm\sqrt{1+4}}{2} = \begin{cases} \frac{-1+\sqrt{5}}{2} \\ \frac{-1-\sqrt{5}}{2}\end{cases}$ $= e^{6-x}(-x^2-x+1) > 0 \iff x^2+x-1<0$

$\iff x \in \left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right) \subseteq (-\infty, 3)$

2

	$\left(-\infty, \frac{-1-\sqrt{5}}{2}\right)$	$\left(\frac{-1-\sqrt{5}}{2}, \frac{-1+\sqrt{5}}{2}\right)$	$\left(\frac{-1+\sqrt{5}}{2}, 3\right)$	$(3,\infty)$
f'	$-$	$+$	$-$	$+$
f	$\searrow$	$\nearrow$	$\searrow$	$\nearrow$

$x=3$: $f'_+(3) \overset{\text{spojitost, V. o lim. der.}}{=} \lim_{x\to 3_+} f'(x) =$

2 $= \lim_{x\to 3_+} e^x(x^2+5x+5) = e^3(9+15+5) = \underline{29e^3}$

$f'_-(3) = \lim_{x\to 3_-} f'(x) = \lim_{x\to 3_-} e^{6-x}(-x^2-x+1) = \underline{\underline{-11e^3}}$

Lokální minima:

$f\left(\frac{-1-\sqrt{5}}{2}\right) = (2-\sqrt{5})e^{\sqrt{5}/2+13/2} \doteq$

1 $\doteq -480{,}3$ (globální min.)

$f(3) = 20\cdot e^3 \doteq 401{,}7$

Lok. maximum:

$f\left(\frac{-1+\sqrt{5}}{2}\right) = (2+\sqrt{5})\cdot e^{\frac{13-\sqrt{5}}{2}} \doteq$

1 $\doteq 921{,}1$

2 $$f''(x) = \left(e^{|x-3|+3}\left(2x+3+\operatorname{sgn}(x-3)(x^2+3x+2)\right)\right)' =$$

$$= e^{|x-3|+3}\cdot \operatorname{sgn}(x-3)\left(2x+3+\operatorname{sgn}(x-3)(x^2+3x+2)\right) +$$

$$+ e^{|x-3|+3}\cdot\left(2+\operatorname{sgn}(x-3)(2x+3)\right) =$$

$$= e^{|x-3|+3}\cdot\left(\operatorname{sgn}(x-3)(4x+6)+x^2+3x+4\right)$$

$\underline{x>3}$: $f''(x) = e^{x}\left(x^2+7x+10\right)$ $\qquad \frac{-7\pm\sqrt{49-40}}{2} = \frac{-7\pm 3}{2} \begin{cases} -2 \\ -5 \end{cases}$

$\underline{x<3}$: $f''(x) = e^{6-x}\left(x^2-x-2\right)$ $\qquad \frac{1\pm\sqrt{1+8}}{2} = \frac{1\pm 3}{2} \begin{cases} 2 \\ -1 \end{cases}$

2

	$(-\infty,-1)$	$(-1,2)$	$(2,3)$	$(3,\infty)$
f''	+	−	+	+
f	∪	∩	∪	∪

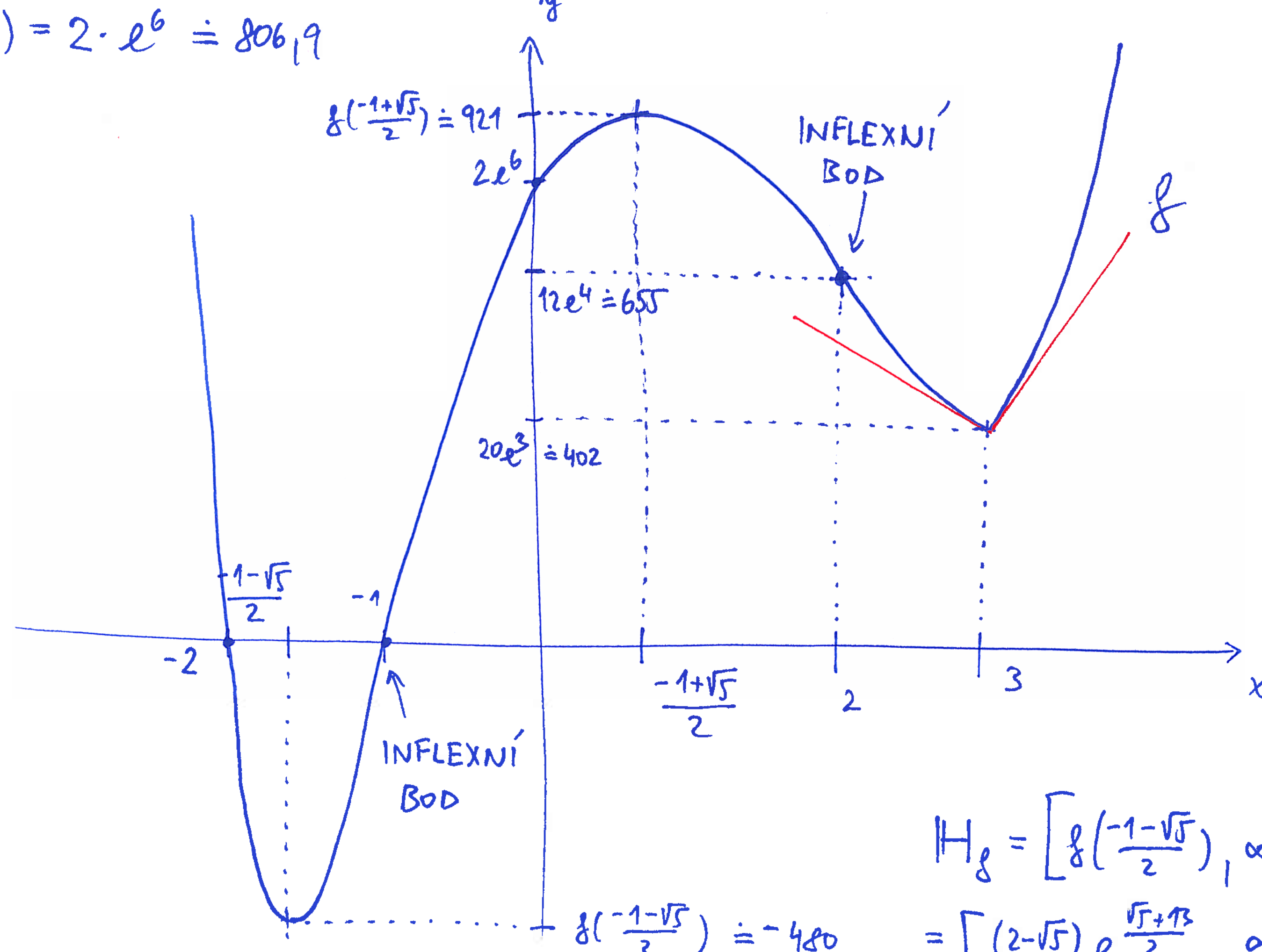
Inflexní body: $\underline{-1, 2}$.

$f(-1)=0$, $f(2) = e^4\cdot 12 \doteq 655{,}2$

$f(x)=0 \iff (x^2+3x+2)=0 \iff (x+1)(x+2)=0 \iff x=-1 \vee x=-2$

$f(0) = 2\cdot e^6 \doteq 806{,}9$

5



$$H_f = \left[f\left(\tfrac{-1-\sqrt5}{2}\right), \infty\right) =$$

$$= \left[(2-\sqrt5)\, e^{\frac{\sqrt5+13}{2}}, \infty\right).$$