

Předmět: NMTM102 Matematická analýza II

Typ výuky: Cvičení

Vahid Borji

borji@karlin.mff.cuni.cz

Cvičení 1 (18.2.2026)

Definice: F je PF k f na $I \Leftrightarrow \forall x \in I: F'(x) = f(x) \quad (F' = f \text{ na } I)$

$$F \stackrel{c}{=} \int f(x)dx \quad F \in \int f(x)dx$$

TABULKA DERIVACÍ

$$(e^x)' = e^x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arctg x)' = \frac{1}{1+x^2}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

TABULKA INTEGRÁLŮ

$$\int e^x dx = e^x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \frac{1}{x} dx = \boxed{\ln|x|} + c$$

$$\int \frac{1}{1+x^2} dx = \arctg x + c$$

$$\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + c$$

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

$$\int \left(\sin x + \frac{1}{1+x^2} \right) dx = \int \sin x dx + \int \frac{1}{1+x^2} dx \stackrel{c}{=} -\cos x + \arctg x$$

ZK: $(-\cos x + \arctg x)' = (-\cos x)' + (\arctg x)' = -(-\sin x) + \frac{1}{1+x^2}$ ✓

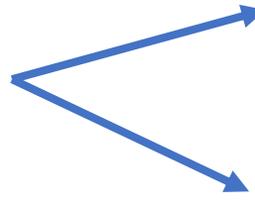
$$\int e^x dx = e^x + c$$

ZK: $(e^x + c)' = e^x$ ✓

$$\int (2\cos x + e^x + \frac{1}{x}) dx = \int 2\cos x dx + \int e^x dx + \int \frac{1}{x} dx =$$

$$2\sin x + e^x + \ln|x| + c \quad \frac{1}{x}$$

ZK: $(2\sin x + e^x + \ln|x| + c)' = 2\cos x + e^x + \cancel{(\ln|x|)'} + 0$

$(\ln|x|)'$ 

$x > 0: (\ln|x|)' = (\ln x)' = \frac{1}{x}$

$x < 0: (\ln|x|)' = (\ln(-x))' = \frac{-1}{-x} = \frac{1}{x}$

$$\alpha \neq -1$$

$$\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + c$$

$$\mathbf{ZK:} \left(\frac{x^{\alpha+1}}{\alpha+1}\right)' = \frac{(\alpha+1)x^\alpha}{\alpha+1} = x^\alpha$$

$$\int (x^2 + 2x + 1) dx = \frac{x^3}{3} + x^2 + x + c$$

$$\int \cos 5x \, dx = \frac{1}{5} \sin 5x + c$$

ZK: $(\frac{1}{5} \sin 5x)' = \frac{1}{5} \cos 5x (5) = \cos 5x$ ✓

$$\int (16e^{2x} + (8x)^3) \, dx = 16 e^{2x} \frac{1}{2} + \frac{1}{4} \frac{(8x)^4}{8} + c = 8e^{2x} + \frac{(8x)^4}{32} + c$$

ZK: $(8e^{2x} + \frac{(8x)^4}{32})' = 8 e^{2x} 2 + \frac{1}{32} 4(8x)^3 8 = 16e^{2x} + (8x)^3$ ✓

$$\int (\sqrt{x} + \sin(3x)) dx = \int \cancel{\sqrt{x}} x^{\frac{1}{2}} dx + \int \sin 3x dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{\cos(3x)}{3} = \frac{2}{3} x^{\frac{3}{2}} - \frac{\cos(3x)}{3} + c$$

$$\int (x + 5)^3 dx = \frac{(x + 5)^4}{4} + c$$

ZK: $((x + 5)^4)' = 4(x + 5)^3$

$$\int \frac{x + 1}{\sqrt{x}} dx = \int \frac{\cancel{x} x^{\frac{1}{2}}}{\cancel{\sqrt{x}}} dx + \int \frac{\cancel{1} x^{-\frac{1}{2}}}{\cancel{\sqrt{x}}} dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = \frac{2}{3} x^{\frac{3}{2}} + 2 x^{\frac{1}{2}} + c$$

$$\int \cos(13x + 17) dx = \frac{\sin(13x + 17)}{13} + c$$

ZK: $\left(\frac{\sin(13x + 17)}{13}\right)' = \frac{1}{13} \cos(13x + 17) (13) = \cos(13x + 17)$

$$\int \cos(ax + b) dx = \frac{\sin(ax + b)}{a} + c$$

$$a \neq 0$$

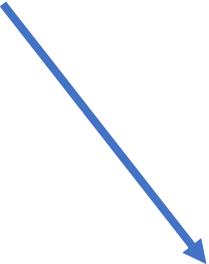
$$\int \frac{1}{2x - 1} dx = \frac{\ln|2x - 1|}{2} + c$$

$$\int \frac{x^2}{1+x^2} dx = \int \frac{x^2+1-1}{1+x^2} dx = \int 1 dx - \int \frac{1}{1+x^2} dx = x - \operatorname{arctg} x + c$$

$$\int \frac{e^{3x}+1}{e^x+1} dx$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$e^{3x} + 1 = (e^x + 1)(e^{2x} - e^x + 1)$$


$$= \int \frac{\cancel{e^x+1} (e^{2x} - e^x + 1)}{\cancel{e^x+1}} dx = \int (e^{2x} - e^x + 1) dx$$

$$= \frac{1}{2} e^{2x} - e^x + x + c$$

$$\int tg^2x dx = \int (1 + tg^2x - 1) dx = \int 1 + tg^2x dx - \int 1 dx = tgx - x + c$$

$$(tgx)' = 1 + tg^2x$$



$$\int (1 + tg^2x) dx = tgx + c$$

$$\int \left(1 - \frac{1}{x^2}\right) \sqrt{\sqrt{x}} dx = \int (1 - x^{-2}) x^{\frac{1}{4}} dx = \int (x^{\frac{1}{4}} - x^{\frac{-7}{4}}) dx =$$

$$\frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1} - \frac{x^{\frac{-7}{4}+1}}{\frac{-7}{4}+1} + c$$