Předmět: NMTM102 Matematická analýza II

Typ výuky: Cvičení

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Cvičení 2 (25.2.2026)

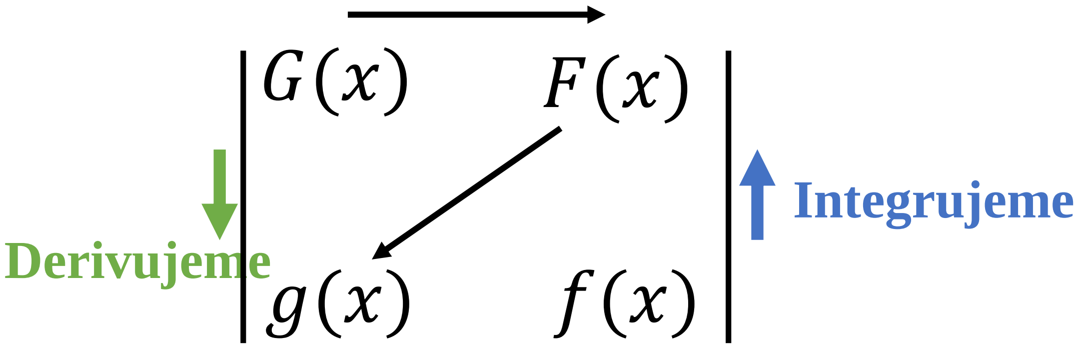
Aplikujeme:

- **Integrace Per Partes**
- **Substituční metoda**

Integrace Per Partes.

Nechť I je otevřený interval, funkce f, g *jsou spojité na* I *a platí* $F' = f$ *a* $G' = g$ *na* I. *Potom:*

$$\int f(x)G(x)\,\mathrm{d}x = F(x)G(x) - \int F(x)g(x)\,\mathrm{d}x \quad na\ I.$$



$$\int xe^x\,dx = ?$$

1)

$$\begin{vmatrix} e^x & \frac{x^2}{2} \\ e^x & x \end{vmatrix}$$

$$\int xe^x\,dx = \frac{x^2}{2}e^x - \boxed{\int \frac{x^2}{2}e^x\,dx}$$

2)

$$\begin{vmatrix} x & e^x \\ 1 & e^x \end{vmatrix}$$

✓

$$\int xe^x\,dx = x\,e^x - \int e^x\,dx \overset{c}{=} xe^x - e^x$$

$$\int x\, sinx\, dx = ?$$

$$\begin{vmatrix} x & -\cos x \\ 1 & sinx \end{vmatrix}$$

$$\int x\, sinx\, dx = -x\cos x - \int -\cos x\, dx = -x\cos x + \int \cos x\, dx$$

$$\overset{c}{=} -x\, cosx + sinx$$

$$\int e^x \, sinx \, dx = e^x \sin x - \int e^x \cos x \;\; dx = e^x \sin x - (e^x cosx - \int (-sinx) e^x dx)$$

$$\begin{vmatrix} sinx & e^x \\ cosx & e^x \end{vmatrix}$$

$$\begin{vmatrix} cosx & e^x \\ -\sin x & e^x \end{vmatrix}$$

$$= e^x sinx - e^x cosx - \int e^x \, sinx \, dx$$

$$\underline{\int e^x \, sinx \, dx} = e^x sinx - e^x cosx - \underline{\int e^x \, sinx \, dx}$$

$$2\int e^x \, sinx \, dx \overset{c}{=} e^x sinx - e^x cosx \Rightarrow \int e^x \, sinx \, dx \overset{c}{=} \frac{e^x sinx - e^x cosx}{2}$$

$$\int e^x \, cosx \, dx = e^x \cos x + \int e^x \, sin\, x \;\; dx = e^x \cos x + e^x \, sinx - \int e^x cosx\, dx$$

$$\begin{vmatrix} cosx & e^x \\ -sinx & e^x \end{vmatrix} \qquad \begin{vmatrix} sinx & e^x \\ \cos x & e^x \end{vmatrix}$$

$$\underline{\int e^x \, cosx \, dx} = e^x \, cosx + e^x sinx - \underline{\int e^x \, cosx \, dx}$$

$$2 \int e^x \, cosx \, dx \stackrel{c}{=} e^x \, cosx + e^x \, sinx \quad \Rightarrow \int e^x \, cosx \, dx \stackrel{c}{=} \frac{e^x \, cosx + e^x \, sinx}{2}$$

$$\int xe^x\, sinx\, dx =$$

$$\left| \begin{array}{cc} x & \dfrac{e^x \sin x - e^x\, cos\, x}{2} \\ 1 & e^x \sin x \end{array} \right|$$

$$\int e^x\, sinx\, dx \overset{c}{=} \frac{e^x \sin x - e^x\, cos\, x}{2}$$

$$\int e^x\, cosx\, dx \overset{c}{=} \frac{e^x \cos x + e^x\, sin\, x}{2}$$

$$x\left(\frac{e^x \sin x - e^x\, cos\, x}{2}\right) - \frac{1}{2}\int (e^x \sin x - e^x\, cos\, x)dx$$

$$\overset{c}{=} \frac{xe^x \sin x - xe^x\, cos\, x}{2} - \frac{1}{2}\left(\frac{e^x \sin x - e^x\, cos\, x}{2}\right) + \frac{1}{2}\left(\frac{e^x \cos x + e^x\, sin\, x}{2}\right)$$

$$= \frac{xe^x \sin x - xe^x\, cos\, x}{2} + \frac{e^x\, cos\, x}{2}$$

$$\int x \sin^2 x \, dx$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\begin{vmatrix} x & \frac{x}{2} - \frac{\sin 2x}{4} \\ 1 & \sin^2 x \end{vmatrix}$$

$$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} dx \overset{c}{=}$$

$$\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)$$

$$= x\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) - \int 1\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) dx$$

$$\overset{c}{=} \frac{x^2}{2} - \frac{x \sin 2x}{4} - \frac{1}{2}\frac{x^2}{2} + \frac{1}{4}(-\cos 2x)\frac{1}{2} = \frac{x^2}{4} - \frac{x \sin 2x}{4} - \frac{1}{8}\cos 2x$$

$$\int \ln x \, dx = \int 1 . \ln x \, dx = x \ln x - \int x \, \frac{1}{x} \, dx \overset{c}{=} x \ln x - x$$

$$\left| \begin{array}{ll} \ln x & x \\ \frac{1}{x} & 1 \end{array} \right|$$

$$\int (\ln x)^2\,dx = \int 1.(\ln x)^2\,dx = x(\ln x)^2 - 2\int \ln x\,dx \overset{c}{=} x(\ln x)^2 - 2(x\ln x - x)$$

$$= x(\ln x)^2 - 2x\ln x + 2x$$

$$\left|\begin{array}{cc} (\ln x)^2 & x \\ 2lnx.\frac{1}{x} & 1 \end{array}\right|$$

$$\int (\ln x)^2\,dx = \int (\ln x)(\ln x)dx = \ln x\,(x\ln x - x) - \int \overset{\ln x - 1}{\cancel{(x\ln x - x)\,\frac{1}{x}}}\,dx$$

$$\overset{c}{=} x(\ln x)^2 - x\ln x - (x\ln x - x - x)$$

$$= x(\ln x)^2 - 2x\ln x + 2x$$

$$\left|\begin{array}{cc} \ln x & x\ln x - x \\ \frac{1}{x} & \ln x \end{array}\right|$$

$$\int \cos(5x)\ dx = \int cost\ \frac{1}{5}dt = \frac{1}{5}\int \cos t\ dt \overset{c}{=} \frac{1}{5}\ sint = \frac{1}{5}\ sin5x$$

$$t = 5x \quad \Rightarrow dt = 5dx$$

$$dx = \frac{1}{5}dt$$

$$\int xe^{x^2}\ dx = \int e^t\ \frac{1}{2}dt \overset{c}{=} \frac{1}{2}e^t = \frac{1}{2}e^{x^2}$$

$$t = x^2 \quad \Rightarrow dt = 2xdx$$

$$xdx = \frac{1}{2}dt$$

$$\int tagx\ dx = \int \frac{sinx}{cosx}\ dx = \int \frac{-dt}{t} \overset{c}{=} -\ln|t| = -\ln|cosx|$$

$$t = cosx \Rightarrow dt = -sinx\ dx$$

$$sinx\ dx = -dt$$

$$\int sin^5x\,dx = \int sin^2x\ sin^2x\,sinx\ dx = \int (1-cos^2x)(1-cos^2x)\ sinx\ \ dx$$

$$= \int (1-cos^2x)^2\ \ sinx\ \ dx\ = \int\ (1-t^2)^2\ \ (-dt)\ = -\int 1-2t^2+t^4\ dt$$

$$t = cosx \quad \Rightarrow dt = -sinx\ dx$$

$$sinx\ dx = -dt$$

$$\overset{c}{=} -(\ t\ \ -2\frac{t^3}{3}\ +\frac{t^5}{5}\ )$$

$$= -(cosx - 2\frac{cos^3x}{3} + \frac{cos^5x}{5})$$

$$\int \frac{arctagx}{x^2+1}dx = \int t\,dt \stackrel{c}{=} \frac{t^2}{2} = \frac{arctag^2 x}{2}$$

$$t = arctagx \Rightarrow dt = \frac{1}{1+x^2}dx$$

$$\int \frac{x}{\sqrt{x^2+5}}dx = \int \frac{\frac{1}{2}dt}{\sqrt{t}} = \frac{1}{2}\int t^{-\frac{1}{2}}dt \stackrel{c}{=} \frac{1}{2}\frac{t^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \sqrt{t} = \sqrt{x^2+5}$$

$$t = x^2+5 \Rightarrow dt = 2xdx$$

$$x\,dx = \frac{1}{2}dt$$

$$\int \frac{x^2}{cos^2(x^3)}dx = \int \frac{\frac{1}{3}dt}{cos^2(t)} \overset{c}{=} \frac{1}{3}\,tagt = \frac{1}{3}\,tag(x^3)$$

$$t = x^3 \Rightarrow dt = 3x^2dx$$

$$x^2dx = \frac{1}{3}dt$$

$$(tagx)' = \frac{1}{cos^2x}$$

$$\int \frac{dx}{x\,lnx} = \int \frac{dt}{t} \overset{c}{=} ln\,|t| = ln\,|\,lnx\,|$$

$$t = lnx \Rightarrow dt = \frac{1}{x}dx$$

$$\int \frac{x^3}{x^8+2}\,dx \qquad \int \sin(\ln x)\,dx \qquad \int \sin(\ln x)\,dx \qquad \int x^5 e^{x^3}\,dx$$

$$\int \frac{dx}{x\ln x\ln(\ln x)} \qquad \int \frac{cosx}{e^x}\,dx$$