Charles University

Faculty of Mathematics and Physics;

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Developing conceptual knowledge in school mathematics

Lesson #5

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The concept of the derivative in different representations

❑ The concept of the derivative in different representations:

We can consider several representations of the concept of the derivative:

- ➢ Algebraic representation [Symbolic representation]
- ➢ Graphical representation
- ➢ Numerical representation

➢ Numerical representation

Approximating the derivative at a point using a table of values of the difference quotient as h approaches zero.

x	1,7	1,8	1,9	2	2,1	2,2	2,3
$f(x)$	4,59	5,04	5,51	6	6,51	7,04	7,59
$x-2$	$-0{,}3$	$-0{,}2$	$-0{,}1$		0,1	0,2	0,3
$f(x)-f(2)$	$-1{,}41$	$-0{,}96$	$-0{,}49$		0,51	1,04	1,59
$\frac{f(x)-f(2)}{x-2}$	4,7	4,8	4,9		5,1	5,2	5,3

$\boldsymbol{f'(2)} \approx ?$

→ **5** ←

Sketch the graph of a function f that satisfies the following conditions:

The function f is continuous

$$f(0) = 2,\ f'(-2) = f'(3) = 0, \text{ a } \lim_{x \to 0} f'(x) = \infty$$

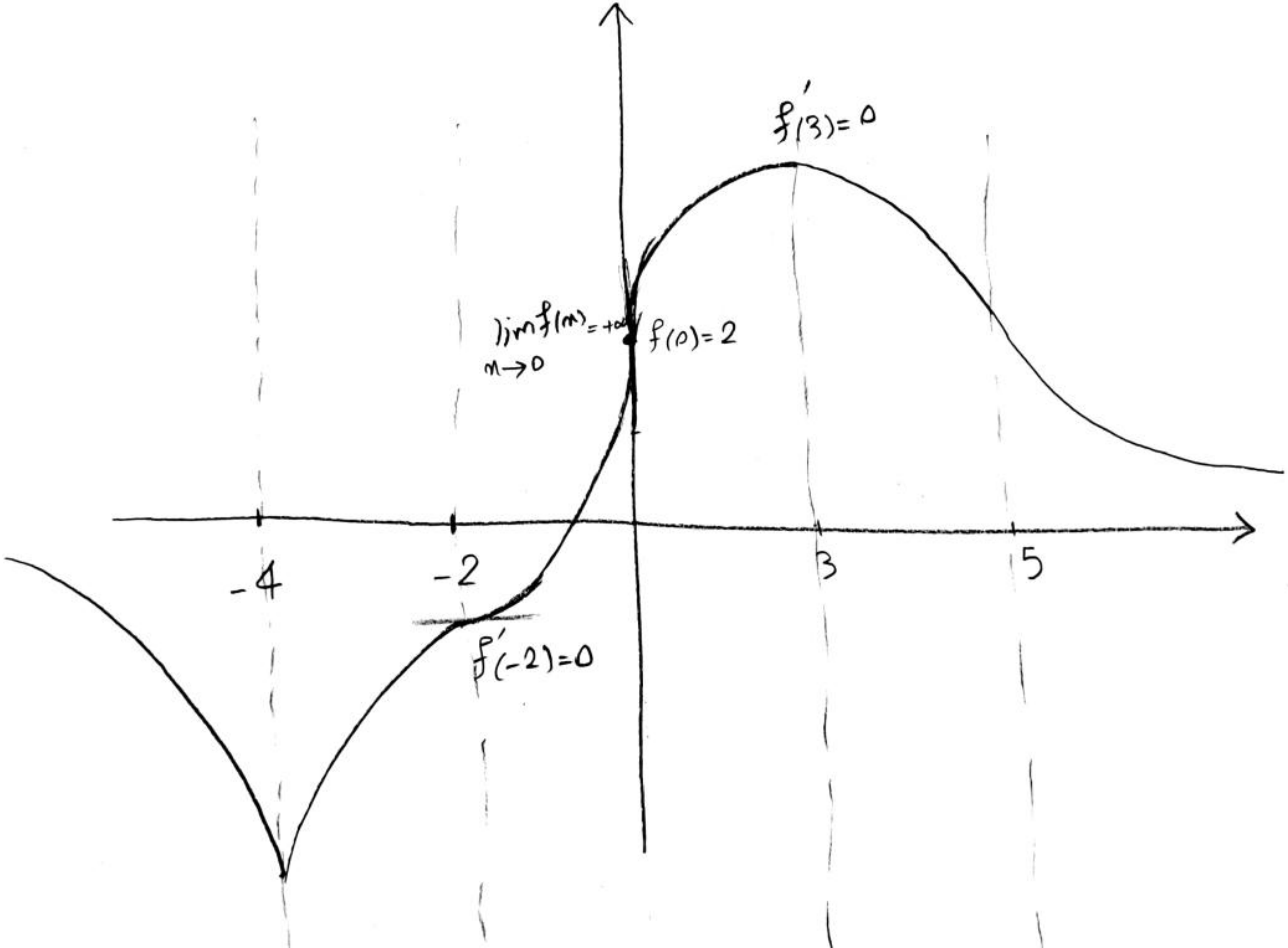
$$f'(x) > 0 \text{ when } -4 < x < -2, \text{ and when } -2 < x < 3,$$

$$f'(x) < 0 \text{ when } x < -4, \text{ and when } x > 3,$$

$$f''(x) < 0 \text{ when } x < -4, \text{ when } -4 < x < -2, \text{ and when } 0 < x < 5,$$

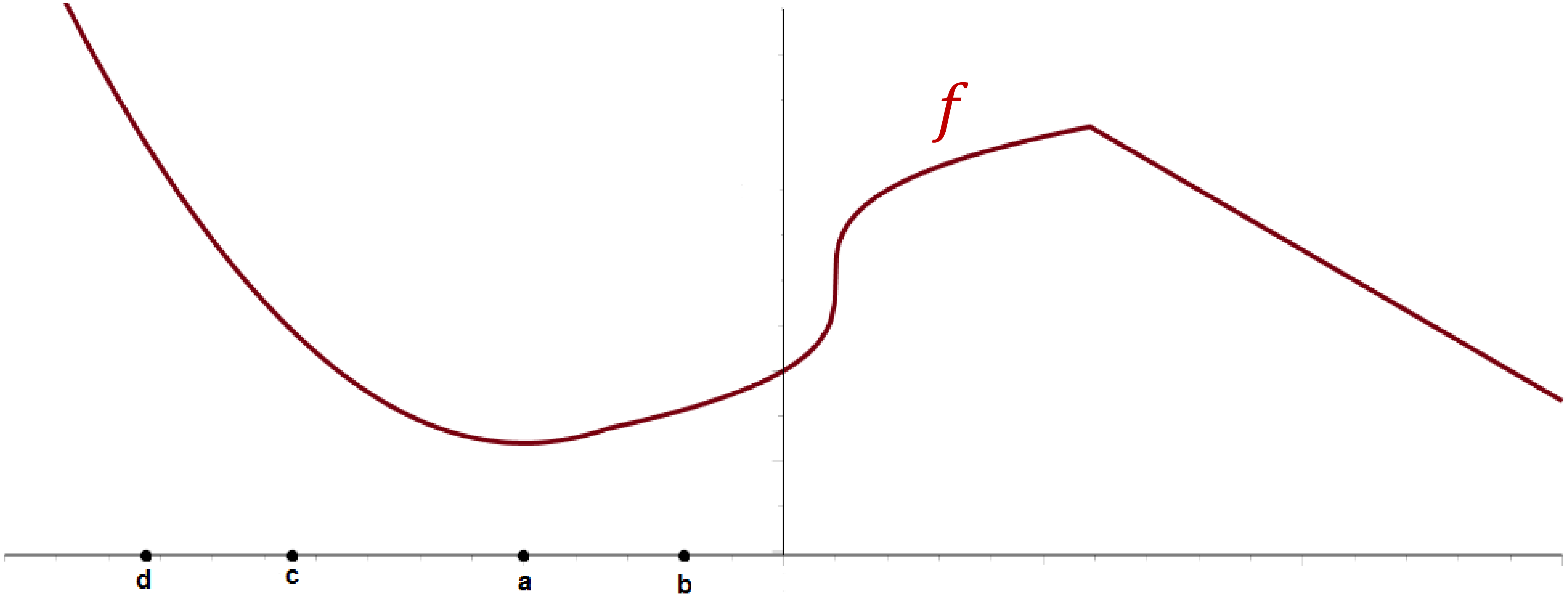
$$f''(x) > 0 \text{ when } -2 < x < 0, \text{ and when } x > 5,$$

$$\lim_{x \to -\infty} f(x) = \infty \text{ and } \lim_{x \to \infty} f(x) = -2$$



f'(3)=0
lim f(x)=+∞
x→0
f(0)=2
-4
-2
3
5
f'(-2)=0

Obrázek níže představuje graf funkce f. Nakreslete graf její derivace.



1. Let $f(x) = \sqrt{x+8} - \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x+8}+\sqrt{x}}$. Compute the value of the expression $f'(1)g(1) - g'(1)f(1)$.

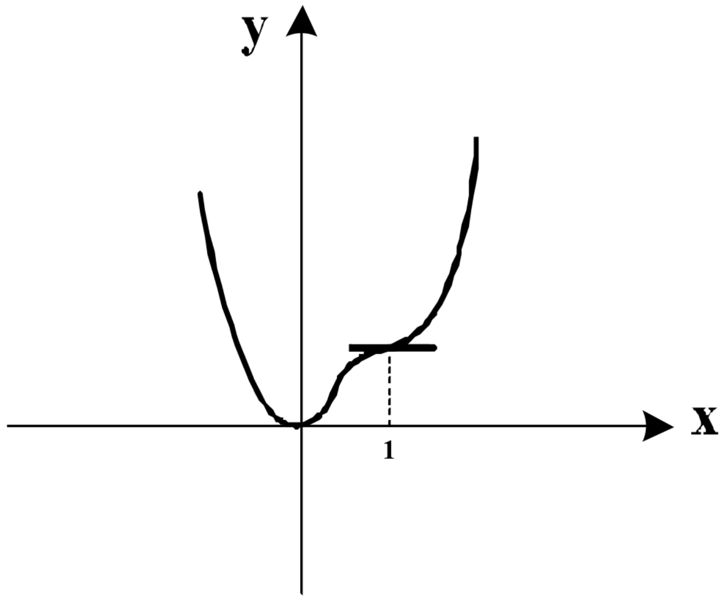
2. For how many integer values of m is the function $f(x) = \frac{mx+2}{x-1+m}$ decreasing on the interval $(1, +\infty)$?

3. For every non-zero real number a, the function $f(x) = \begin{cases} bx + c & x < a \\ \frac{1}{x} & x \geq a \end{cases}$ is differentiable on $\mathbb{R}$. Determine the value of ac.

4. The tangent to the curve $y = x^3 + ax^2 + bx - 1$ at the point $(-1, -4)$ passes through the interior of the graph. Find the value of $\frac{a}{b}$.

5. (Homework) At which point does the tangent to the graph of the function $f(x) = \frac{5x-4}{\sqrt{x}}$ at $x = 4$ intersect the y-axis?

6. The graph of the function $f(x) = 3x^4 + ax^3 + bx^2 + cx$ is given. Determine the value of a.



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Thank you for your attaention

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