Interpretation of exponential and logarithmic functions in contextual situations
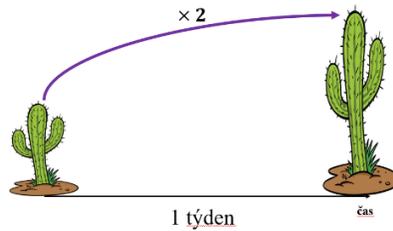
The cactus story: We have a magic cactus, and its height doubles every week.

1. We have a cactus whose height doubles every week. How can we change the base of the logarithm from 2 to another number (for example, 4, 8, $2^{\frac{1}{7}}$, $2^{\frac{365}{7}}$)?

$\log_2 91$ represents the number of weeks the cactus needs to reach 91 times its height.

a) $\log_4 91$ represents the number of ….

b) $\log_8 91$ represents the number of ….

c) $\log_{2^{\frac{1}{7}}} 91$ represents the number of ….

a) $\log_{2^{\frac{365}{7}}} 91$ represents the number of ….

2. How can we change the base of the logarithm from 2 to another number (for example, 3 or 5)?

a) $\log_3 A$ represents the number of ….

b) $\log_5 A$ represents the number of ….

3. We have a cactus whose height doubles every week. Complete each of the following sentences.

a) A doubling period is the time it takes for ….

b) A quadrupling period is the time it takes for ….

c) A tenfold period is the time it takes for ….

4. We have a cactus whose height doubles every week. Answer the following questions [your answer should be a natural number]. For each part, draw a diagram and show how you can use the diagram to justify your answer.

a) How many doubling periods does the cactus need to reach *8 times* its original height?

b) How many quadrupling periods does the cactus need to reach *16 times* its original height?

c) How many sixfold periods does the cactus need to reach *216 times* its original height?

d) How many tripling periods does the cactus need to reach *81 times* its original height?

e) How many fivefold periods does the cactus need to reach *5 times* its original height?

5. We have a cactus whose height doubles every week. Answer the following questions [your answer should be written in the form of a logarithm].

a) How many doubling periods does the cactus need to reach *9 times* its original height?

b) How many quadrupling periods does the cactus need to reach *5 times* its original height?

c) How many sixfold periods does the cactus need to reach *200 times* its original height?

d) How many tripling periods does the cactus need to reach 8 *times* its original height?

6. We have a cactus whose height doubles every week.

a) Correct the following equality by multiplying or dividing one side by a number.

The number of doubling periods the cactus needs to reach *8 times* its original height

=

The number of quadrupling periods the cactus needs to reach 8 times its original height

b) Write the correct equality in the form of a logarithmic equality.

7. We have a cactus whose height doubles every week. Simplify each fraction/ratio.

a) $\dfrac{\textit{The number of doubling periods the cactus needs to reach } 2^k \textit{ times its original height}}{\textit{The number of doubling periods the cactus needs to reach twice its original height}} =$

b) $\dfrac{\textit{The number of quadrupling periods the cactus needs to reach } 2^k \textit{times its original height}}{\textit{The number of quadrupling periods the cactus needs to reach twice its original height}} =$

c) $\dfrac{\textit{The number of fivefold periods the cactus needs to reach } 2^k \textit{ times its original height}}{\textit{The number of fivefold periods the cactus needs to reach twice its original height}} =$

8. Solve the logarithmic equation in the context of the cactus story. Do not solve it algebraically. Draw a diagram of the cactus and solve the problem using this diagram. In your diagram, you must indicate where the variable x and the individual logarithms appear.

$$\log_{10} x + \log_{10} 3 = 2\log_{10} 4 - \log_{10} 2$$

9. Solve the logarithmic equation in the context of the cactus story. Do not solve it algebraically. Draw a diagram of the cactus and solve the problem using this diagram. In your diagram, you must indicate where the variable x and the individual logarithms appear.

$$\log_4(3x + 2) - 2\log_4 x = 2 - \log_4 8$$

10. Homework: We have a cactus whose height doubles every month. Using this cactus, design a simple word problem that demonstrates that the equality $\log_{2^{12}} 17 = \frac{1}{12}\log_2 17$ holds.

Hint: You must answer your word problem in two different ways. In the first method, you must obtain $\log_{2^{12}} 17$ as the answer. In the second method, you must obtain $\frac{1}{12}\log_2 17$. Since the word problem has only one correct answer, you may conclude that the equality $\log_{2^{12}} 17 = \frac{1}{12}\log_2 17$ holds.