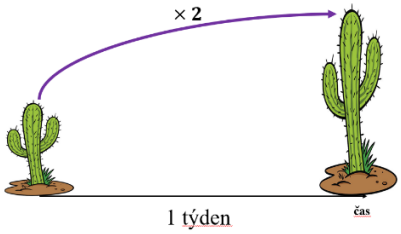
Interpretation of exponential and logarithmic functions in contextual situations

The cactus story: We have a magic cactus, and its height doubles every week.



Pozor: From now on, we will use this context in all examples: we have a magic cactus, and its height doubles every week.

There are two quantities in this story: 1) Height of the cactus 2) Time.

1. What does $\mathbf{2^3}$ mean in the cactus story? (draw a diagram)

2. What does $\mathbf{2^4}$ mean in the cactus story? (draw a diagram)

3. What does $\mathbf{2^{\frac{1}{7}}}$ mean in the cactus story? (draw a diagram)

4. What does $\mathbf{2^0}$ mean in the cactus story?

5. What does $\mathbf{2^{-1}}$ mean in the cactus story?

The meaning of the logarithm in relation to the story of the cactus:

We know $\log_2 8$ is 3, because there are **3 factors of** 2 in the number 8. ($\log_2 8 = \log_2(2 \times 2 \times 2) = 3$)

We know $\log_2 32$ je 5, because there are **5 factors of** 2 in the number 32 ($\log_2 32 = \log_2(2 \times 2 \times 2 \times 2 \times 2) = 5$)

6. How can we describe $\mathbf{\log_2 8}$ in the cactus story? (Draw a diagram and show $\log_2 8$ in your diagram)

7. How can we describe $\mathbf{\log_2 16}$ in the cactus story? (Draw a diagram and show $\log_2 16$ in your diagram)

8. Using the story of the cactus, show why the inequality $\mathbf{\log_2 17 < \log_2 25 < \log_2 31}$ is true (draw a diagram).

Logarithmic rules in terms of the cactus story:

9. Design a word problem (story problem) so that the answer to the problem in the task is $\mathbf{\log_2 3 + \log_2 5}$ and $\mathbf{\log_2 15}$. Then, conclude that the following equality holds:

$$\mathbf{\log_2 3 + \log_2 5 = \log_2 15}$$

10. (Homework) Design a word problem (story problem) so that the answer to the problem in the task is $\mathbf{\log_2 12 - \log_2 4}$ and $\mathbf{\log_2 3}$. Then, conclude that the following equality holds:

$$\mathbf{\log_2 12 - \log_2 4 = \log_2 3}$$

11. Interpret/Describe the equation $\mathbf{\log_2 5^3 = 3 \times \log_2 5}$ in the cactus story about the cactus (draw a diagram.)

12. We have a cactus whose height doubles every week. Fill in the blanks.

a) The number of weeks the cactus needs to reach 2^3-times its height is …

b) The number of weeks the cactus needs to reach 2^4-times its height is …

c) The number of weeks the cactus needs to reach 2^{100}-times its height is …

d) The number of weeks the cactus needs to reach 2^A-times its height is …

e) Write the statement from part (d) as a logarithmic equation.

13. We have a cactus whose height doubles every week. Fill in the blanks.

a) $2^{\text{the number of weeks the cactus needs to reach } 8\text{-times its height}}$ is …

b) $2^{\text{the number of weeks the cactus needs to reach } 9\text{-times its height}}$ is …

c) $2^{\text{the number of weeks the cactus needs to reach } 10\text{-times its height}}$ is …

d) $2^{\text{the number of weeks the cactus needs to reach } 100\text{-times its height}}$ is …

e) $2^{\text{the number of weeks the cactus needs to reach } A\text{-times its height}}$ is …

f) Write the statement from part (e) as a logarithmic equation.

14. Use the cactus story and explain why this these equalities are true.

$$\frac{\log_{\mathbf{2}} \mathbf{15}}{\log_{\mathbf{2}} \mathbf{5}} = \frac{\log_{\mathbf{4}} \mathbf{15}}{\log_{\mathbf{4}} \mathbf{5}} = \log_{\mathbf{5}} \mathbf{15}$$