Recommended Problems 7 - Solutions

7.1 Assume G has a perfect matching M.

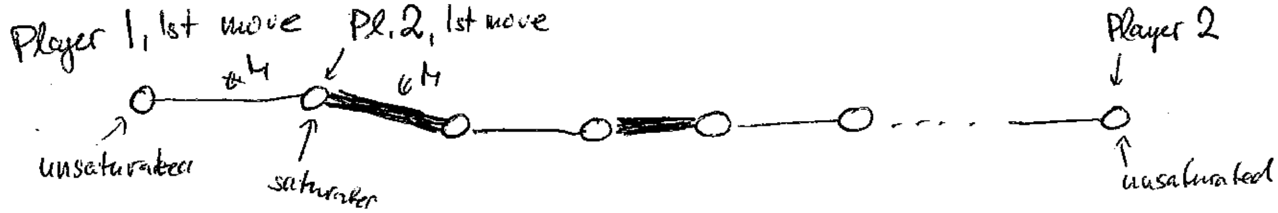
Winning strategy for Player 2: If the last vertex selected by Player 1 was v then choose the vertex w such that $v \leftrightarrow w \in M$

The strategy works because after each move of Player 2 the set of used vertices is equal to the set of all endpoints in a subset of M. (Therefore Player one is forced to select an endpoint of $m \in M$ where the other endpoint is not yet used.)

Assume M is a maximum matching of G, M is not perfect.

Winning strategy for Player 1: 1. Choose an unsaturated vertex and then follow the same strategy as above (i.e. if the last vertex selected by Player 2 was v then choose the vertex w such that $v \leftrightarrow w \in M$)

The argument is similar as above. We need to observe that Player 2 always selects a saturated vertex. Indeed, if not then we obtain an M-augmenting path which is impossible as M is maximum:



7.2 A necessary and sufficient condition for being able to fill all trips is:

For every subset $T \subseteq \{t_1, \dots, t_n\}$ the number of people which like at least one of the trips in T is at least $\sum_{i;\, t_i \in T} c_i$.

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7.2 contd) It can be seen from the following X,Y-bigraph:

$$X = \{ t_1^{(1)}, \dots t_1^{(c_1)}, t_2^{(1)}, \dots, t_2^{(c_2)}, \dots, \dots, t_n^{(c_n)} \}$$

(i.e. c_i vertices for each trip t_i)

Y = people

$t_i^{(j)} \leftrightarrow g$ iff g likes t_i

The requiered condition is equivalent to existence of a matching which saturates X. Hall's condition for this graph is equivalent to the condition stated in the beginning. Details are left for the reader.

7.3) Every vertex of G covers at most $\Delta(G)$ edges, so a set S covers at most $|S|\cdot\Delta(G)$ edges. Therefore, if S covers all $(=|E(G)|)$ edges, then $|S| \geq |E(G)|/\Delta(G)$, and the minimum size of a vertex cover is greater than or equal to $|E(G)|/\Delta(G)$. The claim now follows from the König – Egerváry Theorem.

7.4) (a) Discussed in class

(b) Using part (a) ~~we can rearran~~ (multiple times) we can rearrange the cards inside each column so that every row in our array contains all values (exactly once). Then, by switching cards of the same value, we can obtain an arrangement where the first row is monochromatic (=all the cards in the first row have the same suit). Then we can make the second row monochromatic, etc... Details are again left for the reader.