Recommended Problems 7

(7.1) Two people play a game on a graph G, alternately choosing distinct vertices. Player 1 starts by choosing any vertex. Each subsequent choice must be adjacent to the preceding choice (of the other player). Thus together they follow a path. The last player able to move wins.

Prove that the 2nd player has a winning strategy if G has a perfect matching and otherwise the 1st player has a winning strategy.

(7.2) The people in a club are planning their summer vacations. Trips $t_1, \ldots, t_n$ are available, but t_i has capacity c_i. Each person likes some of the trips and will travel on at most one. In terms of which people like which trips, derive a necessary and sufficient condition for being able to fill all trips (to capacity) with people who like them.

(7.3) Use the König-Egerváry Theorem to prove that every bipartite graph G has a matching of size at least $|E(G)|/\Delta(G)$.

(7.4) A deck of $m \cdot n$ cards with m values and n suits consists of one card of each value in each suit. The cards are dealt into an $n \times m$ array.

(a) Prove that there is a set of m cards, one in each column, having distinct values.

(b) Use part (a) to prove that by a sequence of exchanges of cards of the same value, the cards can be rearranged so that each column consists of n cards of distinct suits.