Recommended Problems 3

(3.1) Let G be a simple graph having no isolated vertex and no induced subgraph with exactly two edges. Prove that G is a complete graph.

(3.2) Let G be a connected simple graph not having P_4 or C_3 as an induced subgraph.

(3.3) Prove that every $\underline{n}$-vertex graph with at least $\underline{n}$ edges contains a cycle.
(Hint: use induction and the fact that "all degrees ≥ 2" $\Rightarrow$ "cycle")

(3.4) <u>Def.</u> The <u>union</u> of graphs $G_1, \ldots, G_k$, written $G_1 \cup \ldots \cup G_k$, is the graph with vertex set $V(G_1) \cup \ldots \cup V(G_k)$ and edge set $E(G_1) \cup \cdots E(G_k)$.

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a) Given $n \leq 2^k$, encode the vertices of K_n as distinct binary k-tuples. Use this to construct $\underline{k}$ bipartite graphs whose union is K_n.

b) Given that K_n is a union of bipartite graphs $G_1, \cdots, G_k$, encode the vertices of K_n as distinct binary k-tuples. Use this to prove that $n \leq 2^k$.

These two steps prove the following theorem

<u>Theorem</u> The complete graph K_n can be expressed as the union of $\underline{k}$ bipartite graphs if and only if $n \leq 2^k$