

## Recommended problems 1

- (1.1) Determine the number of isomorphism classes of simple 7-vertex graphs in which every vertex has degree 4.
- (1.2) Let  $G$  be a graph with girth 5. Prove that if every vertex of  $G$  has degree at least  $k$ , then  $G$  has at least  $k^2+1$  vertices. For  $k=2$  and  $k=3$ , find one such graph with exactly  $k^2+1$  vertices.
- (1.3) Prove that a self-complementary graph with  $n$  vertices exists if and only if  $n$  or  $n-1$  is divisible by 4. (Hint: When  $n$  is divisible by 4, generalize the structure of  $P_4$  by splitting the vertices into four groups. For  $n \equiv 1 \pmod{4}$ , add one vertex to the graph constructed for  $n-1$ .)
- (1.4) Let  $G$  be a simple graph in which each vertex has degree 3. Prove that  $G$  decomposes into claws if and only if  $G$  is bipartite.

