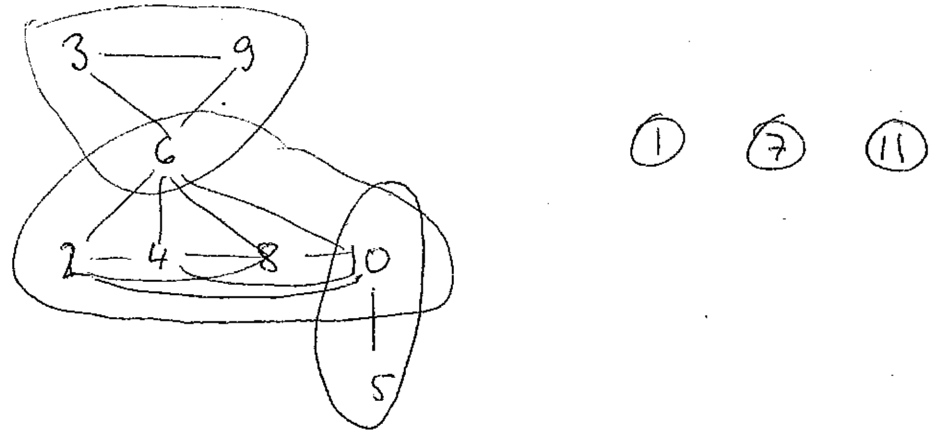
Homework 8 – Solutions

8.1



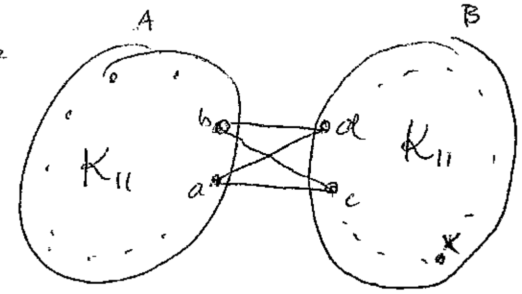
The blocks of G are the subgraphs induced by $\{1\}, \{7\}, \{11\}$, $\{5,10\}$, $\{3,6,9\}$, $\{2,4,6,8,10\}$.

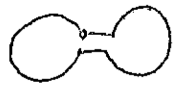
a cut-edge

isolated vertices

both 2-connected
and 6 is a cut vertex, so they are maximal 2-connected

8.2 For instance

$G =$ 

- every vertex has degree ≥ 10 and $d(x) = 10$ so $\boxed{\delta(G) = 10}$
- $\kappa(G) \leq 2$ because $\{a,b\}$ is a vertex cut
- $\kappa(G) > 1$ because there is no cut vertex (this can be seen, for instance, from the fact that G has a spanning cycle )

$\rightarrow \boxed{\kappa(G) = 2}$

- $\kappa'(G) \leq 4$ because $\{ac, ad, bc, bd\}$ is a disconnecting set of edges
- $\kappa'(G) > 3$. Take away 3 edges arbitrarily. The subgraph induced by A (and B) remains connected as $\kappa'(K_{11}) > 3$. Also ~~A and B are connected~~ because at least one of the edges ac, ad, bc, bd remained.

A is connected to B

$\rightarrow \boxed{\kappa'(G) = 4}$

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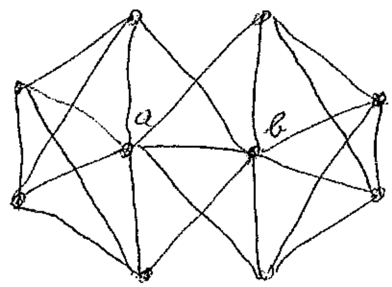
8.3

- $\boxed{\delta(G)=4}$ (the minimum degree)



- $\kappa(G) \leq 2$ because $\{a, b\}$ is a vertex cut
- $\kappa(G) > 1$ because there is no cut vertex (as G has a spanning cycle

)

→ $\boxed{\kappa(G)=2}$

- $\kappa'(G) \leq 4$ because $\kappa'(G) \leq \delta(G)$
- $\kappa'(G) > 3$. Assume that $\kappa'(G) \leq 3$ and let $[S, S']$ be an edge cut with $|[S,S']| \leq 3$. From class we know that in this case $|S| > \delta(G)$ and $|\bar{S}| > \delta(G)$, so $|S| = |\bar{S}| = 5$. One of the sets contains a, WLOG assume $a \in S$. Then

$$|[S,\bar{S}]| = \underbrace{\sum_{v\in S} d(v)}_{\substack{\geq \\ 4+4+4+4+6}} - 2\underbrace{|E(G[S])|}_{\substack{\leq \\ 9}} \geq 4, \text{ a contradiction}$$

(as $\delta(G) \geq 4$ and $a \in S$)

9 (as G doesn't contain K_5 * so at least one edge is missing)

* this can be seen by inspecting neighborhoods of vertices of degree 4 (there is always at least one edge missing in the neighborhood)

→ $\boxed{\kappa'(G)=4}$