Homework 4

(4.1) Let G be a k-regular simple graph with n vertices. Determine the number of subgraphs of G isomorphic to P_3.

(4.2) For each $k \geq 3$, determine the smallest n such that

a) there is a simple k-regular graph with n vertices.

b) there exist nonisomorphic simple k-regular graphs with n vertices.

(Hint: for b) consider complements)

(4.3) Let $d_1, \ldots, d_n$ be integers such that $d_1 \geq \ldots \geq d_n \geq 0$. Prove that there is a loopless (multiple edges allowed) graph with degree sequence $d_1, \ldots, d_n$ if and only if $\sum_{1 \leq i \leq n} d_i$ is even and $d_1 \leq d_2 + \ldots + d_n$.