

## Homework 5

Deadline 5.01.2023, 17:20

1. (10 points) Let  $\mathcal{C} = \text{Clo}(\mathbf{A})$ , where  $\mathbf{A} = (\{1, 2, 3, 4\}, *)$  with

$*$	1	2	3	4
1	2	3	2	1
2	1	4	3	4
3	2	1	2	1
4	3	4	3	2

- (a) Prove that there is no 5-ary operation  $f \in \mathcal{C}$  satisfying  $f(4, 2, 4, 4, 4) = 1$
- (b) Prove that there is no 5-ary operation  $f \in \mathcal{C}$  satisfying  $f(2, 1, 3, 4, 3) = 1$  and  $f(2, 1, 1, 4, 3) = 2$
2. (10 points) Let  $\mathbf{L} = (\{0, 1, 2\}, \wedge, \vee)$  be the three-element lattice. Find a monotone idempotent operation that is not in  $\text{Clo}(\mathbf{L})$ .
3. (10 points) In the lecture you saw that  $\mathcal{C} = \text{Pol}(\text{Inv}(\mathcal{C}))$ , if  $\mathcal{C}$  is a clone on some finite set  $A$ . Show that for clones on an infinite  $A$  this is not true in general (Hint: study the clone generated by all bijections  $A \rightarrow A$ ).