

Homework 2

Deadline 10.11.2022, 17:20

1. (10 points) Determine all the subalgebras and congruences of $(\mathbb{N}, *)$ where $x * y = \max(x, y) + 1$. Draw the lattices Sub and Con.
2. (10 points) Let $\mathbf{G} = (G, \cdot, {}^{-1}, e)$ be a group. Prove that there is a lattice isomorphism between the lattice of normal subgroups of \mathbf{G} and the lattice of congruences of \mathbf{G} .
3. (10 points) For a fixed prime p consider the algebra $\mathbf{A} = (\{0, 1, \dots, p-1\}, m)$, where m is a ternary operation defined by $m(x, y, z) = x - y + z \pmod{p}$. Prove that for any n , R is a subuniverse of \mathbf{A}^n if and only if R is empty or an affine subspace of \mathbb{Z}_p^n . (Recall from linear algebra that R is an affine subspace iff it is closed under all affine combinations.)