Universal Algebra 1 – Exercises 11

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Exercise 1. Let $<$ be the *strict less-than* order relation on $\{0,1\}$. Prove or disprove the following.

(i) $\mathrm{Pol}(<) = \mathrm{Pol}(\{0\},\{1\})$.

(ii) $\mathrm{Pol}(<) = \mathrm{Pol}(\{0,1\})$.

Exercise 2. An n-ary operation f on a set A is called *conservative* if for all elements $a_1,\dots,a_n \in A$ we have $f(a_1,\dots,a_n) \in \{a_1,\dots,a_n\}$. Let A be a set and denote by $\mathcal{C}$ be the set of all conservative operations on A.

(i) Prove that $\mathcal{C}$ is a clone on A.

(ii) Prove that $\mathcal{C} = \mathrm{Pol}(\mathrm{Rel}_1(A))$.

Exercise 3. Let f be a *boolean function*, that is, $f\colon \{0,1\}^n \to \{0,1\}$ for some $n \in \mathbb{N}$. Prove that the following are equivalent.

(i) There is an index $i \in \{1,\dots,n\}$ such that $f(x_1,\dots,x_n) \geq x_i$ for every $x_1,\dots,x_n \in \{0,1\}$.

(ii) The function f preserves the relation $R_k = \{0,1\}^k \setminus \{(0,\dots,0)\}$ for every $k \in \mathbb{N} \setminus \{0\}$.

Exercise 4. Let A be a set and define

$$\nu = \{(x,y,z) \in A^3 \colon x = y \text{ or } y = z\}.$$

Prove that the clone of essentially unary operations, $\mathcal{U}(A)$, is equal to $\mathrm{Pol}(\nu)$.