

Universal Algebra 1 – Exercises 10

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8 December 2022, Prague

Exercise 1. Consider the binary operation \rightarrow on $2 = \{0, 1\}$ defined by

| | | |
|---------------|---|---|
| \rightarrow | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |

- (i) Prove that $\vee \in \text{Clo}(\langle 2, \rightarrow \rangle)$.
- (ii) Determine and write the Cayley table of $\text{Clo}_n(\langle 2, \rightarrow \rangle)$ for $n = 1, 2$.
- (iii) Prove that an n -ary operation f is a member of $\text{Clo}(\langle 2, \rightarrow \rangle)$ if and only if there is $i \leq n$ such that $f(x_1, \dots, x_n) \geq x_i$.

Exercise 2. Let $\mathbf{A} = \langle \{0, 1, 2\}, \cdot \rangle$ be the binar with Cayley table

| | | | |
|---------------|---|---|---|
| \rightarrow | 0 | 1 | 2 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 2 | 0 | 0 | 2 |

- (i) Determine and write the Cayley table of $\text{Clo}_n(\mathbf{A})$ for $n \leq 3$.
- (ii) Prove that $|\text{Clo}_n(\mathbf{A})| = n + 2^n - 1$.

Exercise 3. Let A be a finite set. Prove that $\text{Clo}^A(\text{Op}_2(A)) = \bigcup_{n>0} \text{Op}_n(A)$.