Universal Algebra 1 – Exercises 1

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29 September 2022, Prague

Exercise 1.

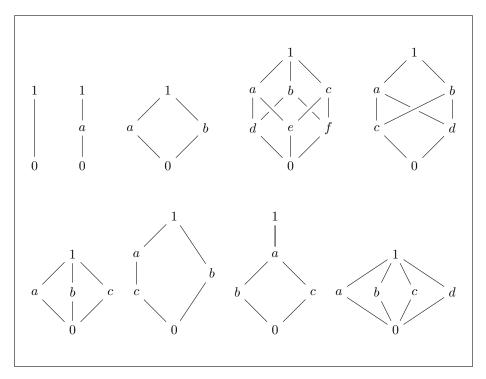
- (i) Let $\mathbf{S} = \langle S, \wedge \rangle$ be a semilattice. Find and define a partial order relation \leq on S such that $\langle S, \leq \rangle$ is a semilattice ordered set.
- (ii) Let $\langle S, \leq \rangle$ be a semilattice ordered set. Find and define a binary operation \wedge on S such that $\langle S, \wedge \rangle$ is a semilattice.
- (iii) Verify that the composition of the constructions (i) and (ii) acts as the identity on the semilattice $\mathbf{S} = \langle S, \wedge \rangle$.
- (iv) Verify that the composition of the constructions (ii) and (i) acts as the identity on the semilattice ordered set $\langle S, \leq \rangle$.
- (v) Conclude that the definitions of semilattice and semilattice ordered set are equivalent.

Exercise 2.

- (i) Let $\mathbf{L} = \langle L, \wedge, \vee \rangle$ be a lattice. Find and define a partial order relation \leq on L such that $\langle L, \leq \rangle$ is a lattice ordered set.
- (ii) Let $\langle L, \leq \rangle$ be a lattice ordered set. Find and define two binary operations \wedge and \vee on L such that $\langle L, \wedge, \vee \rangle$ is a lattice.
- (iii) Verify that the composition of the constructions (i) and (ii) acts as the identity on the lattice $\mathbf{L} = \langle L, \wedge, \vee \rangle$.
- (iv) Verify that the composition of the constructions (ii) and (i) acts as the identity on the lattice ordered set $\langle L, \leq \rangle$.
- (v) Conclude that the definitions of lattice and lattice ordered set are equivalent.

Exercise 3. Are there isomorphic algebras among the following families?

- (i) $\langle \mathbb{Q}, +, 0 \rangle$, $\langle \mathbb{R}, +, 0 \rangle$
- (ii) $\langle \mathbb{N}, \cdot \rangle$, $\langle 2\mathbb{N}, \cdot \rangle$, $\langle 3\mathbb{N}, \cdot \rangle$, $\langle 2\mathbb{N}+1, \cdot \rangle$
- (iii) $\langle \mathbb{C}, + \rangle$, $\langle \mathbb{R}, + \rangle \times \langle \mathbb{R}, + \rangle$
- (iv) $\langle \mathbb{C}, \cdot \rangle$, $\langle \mathbb{R}, \cdot \rangle \times \langle \mathbb{R}, \cdot \rangle$



Exercise 4. Which of the following Hasse diagrams represent a lattice?