

UA2 Practical 6

→ may assume idempotent finite

① Let $R \leq_{sd} \underline{A}^2$, $\underline{B} \triangleleft_t \underline{A}$, $S = R \cap (B \times B)$.

Prove that (subdirect part of S) $\triangleleft_t \underline{A}$.

② Let \underline{A} idempotent finite, $R \leq_{sd} \underline{A} \times \underline{A}$ linked, $\underline{B} \triangleleft \underline{A}$, $S = R \cap (B \times B) \leq_{sd} \underline{B} \times \underline{B}$.

Prove that S is linked.

binary operation

③ Prove that $\text{Clo}(\{\text{rock, paper, scissors}\}; \text{winner}\})$ is a minimal Taylor clone.

(Hint: Look at how operations behave on 2-element subsets)

④* Prove that a ternary (n -ary, $n \geq 3$) $R \leq_{sd} A \times A \times A$ which is irredundant pp-defines ~~either~~

- binary $S \leq_{sd} A \times A$ which is irredundant or

- ternary $T \leq_{sd} A \times A \times A$ which is strongly functional

Prove "strongly functional \Rightarrow abelian" and deduce

Zhuk's 4-types theorem (with absorption instead of 3-abs.)

(\square and "no subdirect irredundant \Rightarrow polynomially complete")

⑤ In the proof of Hell-Nesetril's theorem (12.6) what if S is not connected?