UA 2 Practical 5

① Let $R \leq_{sd} \underline{A} \times \underline{B}$, where $\underline{A}, \underline{B}$ are finite, simple. Show that either R is the graph of a bijection $A \to B$ or R is linked

② Let $\underline{A}, \underline{B}$ be finite idempotent, $C \subseteq A$, $D \subseteq B$, $R \leq_{sd} C \times D$ linked. Prove that $Sg_{\underline{A} \times \underline{B}}(R) \leq_{sd} Sg_{\underline{A}}(C) \times Sg_{\underline{B}}(D)$ is linked.

③ Let $\underline{A}$ be finite and idempotent. Prove that $\forall a \in A \ \{a\} \trianglelefteq \underline{A}$ iff $Clo(\underline{A})$ contains a near-unanimity operation

④ Let $\underline{A}$ be finite Taylor, $\alpha, \beta \in Con\,\underline{A}$ such that $\alpha \wedge \beta = 0$ and $\alpha \vee \beta = 1$. Prove that $\alpha \circ \beta = 1$ $(= \beta \circ \alpha)$ or $\underline{A}$ contains a proper absorbing subalgebra.
(hint: look at A "$\leq$" $A/\alpha \times A/\beta$)

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⑤ Prove that $B \leq \underline{A}$ is projective iff $\forall n \ R_n \leq \underline{A}^2$, where $R_n(x_1, \dots, x_n)$ iff $B(x_1) \vee \dots \vee B(x_n)$ (i.e. $R = A^n \setminus (A \setminus B)^n$)

⑥ Assume that $\underline{A}$ is idempotent and no subalgebra of $\underline{A}$ has a proper 2-absorbing subalgebra.
Prove that $Clo(\underline{A})$ contains a cube operation.