UA2 Practical 4

① Prove that for finite idempotent $\underline{A}$

$\underline{Naked}_2 \in HSP(\underline{A}) \Rightarrow \underline{Naked}_2 \in HS\cancel{P}(\underline{A})$

every operation is a projection
2-element universe

- recall $\underline{Naked}_2 \in HSP_{fin}(\underline{A})$
- Say $A = \{1,2,3,4,\dots\}$, $R = \{11, 12, 13, 34, \underset{\cdots}{?,?}\dots\} \leq \underline{A}^2$
  $\sim = \dots 11, 12, 34 \mid 13 \dots \in Con(\underline{R})$, $\underline{R}/\sim$ naked
  why naked $\in HS(\underline{A})$?
- Say $A = \{1,2,3\}$, $R = \{13, 23, 21, 32\} \leq \underline{A}^2$
  $\sim = 13, 23, 21 \mid 32 \in Con(\underline{R})$, $\underline{R}/\sim$ naked
  why naked $\in HS(\underline{A})$?
- prove the implication for HSP_2 & generalize

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② Take ~~$\underline{A} = (\{n\}; f)$ $f(i) := i+1$~~

$\underline{A} = (\{0,1,\dots,4\}; f)$ $f(i) := i+1 \mod 5$

- what is $Clo(\underline{A})$?
- ~~constr~~ prove that $\underline{naked}_2 \in HSP(\underline{A})$

bonus * explicitly construct $\underline{naked}_2 \in HSP(\underline{A})$

- prove that $\underline{naked}_2 \notin HS\cancel{P}(\underline{A})$
- find a simpler proof that for finite $\underline{A}$

$naked_2 \in HSP(\underline{A}) \not\Rightarrow \underline{naked}_2 \in HS\cancel{P}(\underline{A})$

(42)

(3) Take $\underline{A} := (\mathbb{N};$ all injective operations$)$

- What is $Clo(\underline{A})$? $\quad \mathcal{A} := Clo(\underline{A})$
- Prove that $\mathcal{A}$ contains no Taylor operation
- Show that $\exists f, u \in \mathcal{A}$ $\quad f(y,x) \approx u(f(x,y))$

ie. $\mathcal{A} \not\to \mathrm{Proj} \not\Rightarrow \mathcal{A}$ contains Taylor

bonus * Find nontrivial height 1 identities satisfied by $\mathcal{A}$

ie $\mathcal{A} \not\xrightarrow{min} \mathrm{Proj} \not\Rightarrow \mathcal{A}$ contains Taylor

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(4) Prove that if $\mathcal{A}$ contains a p-ary cyclic operation for every sufficiently large prime p, then $\mathcal{A}$ has a 4-ary Siggers operation $s(r,a,r,e) \approx s(a,r,e,a)$

(hint: identify variables in cyclic operation of suitable arity)

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real interval

(5) Take $\mathcal{A} := \{ f : (0,1)^n \to (0,1) ;$ f acts like a projection in the limit to $\{0,1\}^n$ and f is idempotent $\}$

- formalize the "definition" and show that $\mathcal{A}$ is a clone
- Find a homomorphism $\mathcal{A} \to \mathrm{Proj}$ $(\Rightarrow \mathrm{Proj} \in HSP(\mathcal{A}))$
- Prove that naked $\notin HS(\mathcal{A})$

$\Rightarrow$ for idempotent $\mathrm{Proj} \in HSP(\mathcal{A}) \not\Rightarrow \mathrm{Proj} \in HS(\mathcal{A})$