UA2 Practical 3

1. Let $\underline{L}$ be a distributive lattice. Prove that $(c,d) \in Cg_{\underline{L}}(a,b)$ iff $c \wedge (a \wedge b) = d \wedge (a \wedge b)$ and $c \vee (a \vee b) = d \vee (a \vee b)$.

   Deduce that the variety of distributive lattices has DPC

2. Let $\mathbb{A} = (A; R_1, \dots, R_k)$ be a finite relational structure and $\mathcal{R} = \{R_1, \dots, R_k\}$. Show that $\mathrm{Hom}(\mathbb{A}) \sim CSP(\mathcal{R})$

   $\boxed{\mathrm{Hom}(\mathbb{A})}$

   INPUT: $\mathbb{X}$ finite relational structure similar to $\mathbb{A}$

   QUESTION: is there a homomorphism $\mathbb{X} \to \mathbb{A}$?

3. Prove that for every finite structure $\mathbb{A}$ there exists a finite $\mathbb{B}$ such that

   (i) $\exists$ homomorphisms $\mathbb{A} \to \mathbb{B}$ and $\mathbb{B} \to \mathbb{A}$

   (ii) every endomorphism of $\mathbb{B}$ is an automorphism of $\mathbb{B}$.

   Moreover, such a $\mathbb{B}$ is unique up to isomorphism

4. Assume that all relations in $\mathcal{R}$ are subdirect and $\mathrm{Pol}(\mathcal{R})$ contains a semilattice operation.

   Show that every instance of $CSP(\mathcal{R})$ is a YES-instance

5.* Prove that $CSP(\mathcal{R})$ is in P whenever $\mathrm{Pol}(\mathcal{R})$ contains a semilattice operation